Engineering Structures and Technologies
ISSN 2029-882X / eISSN 2029-8838
2021 Volume 13 Issue 1: 26–30
https://doi.org/10.3846/est.2021.18404

A SIMPLE APPROACH FOR MULTI-CRITERIA DECISION-MAKING ON BASIS OF PROBABILITY THEORY

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Received 21 August 2021; accepted 21 October 2022

Abstract. In the present paper, a new concept of favorable probability is proposed first, and a simplified approach for multi-criteria decision-making is developed on basis of probability theory. It assumes that all the performance indexes of all alternatives can be divided into beneficial and unbeneficial types in the multi-criteria decision-making, and each performance index of all alternative makes its contribution to a partial favorable probability in positively or negatively correlative manners linearly upon its type of beneficial or unbeneficial; the partial favorable probability of each performance index with the same physical meaning is normalized in the alternative group; the product of all partial favorable probability of an alternative makes the total favorable probability of the alternative. As a consequence, all the alternatives can be ranked according to their total favorable probability comparatively in the multi-criteria decision-making. As application examples of the new method, the quantitative assessment of multi-criteria selection for effective dwelling house walls, project managers and contractor for construction works are given in detail, satisfied results are obtained.

Keywords: quantitative assessment, favorable probability, probability based approach, multi-criteria decision-making, overall consideration.

Introduction

In the past decades, multi-criteria decision-making (MCDM) technique becomes an important subject in many fields for decision-making from social life to scientific research, from engineering to economy, which involves the subjective evaluation of performance criteria by decision-makers and detailed mathematical algorithm (Zavadskas et al., 2014). A lot of investigation was conducted to develop detailed mathematical algorithms for assessment of MCDM (Zavadskas & Turskis, 2010; Zavadskas et al., 2012).

Recently, several studies have been focused to solve MCDM problems in buildings and constructions (Şengül et al., 2015; Soltani et al., 2015; Zavadskas et al., 2015), and project management (Monghasemi et al., 2015).

Zavadskas et al. (Zavadskas et al., 2008a) developed "specific method of multiple criteria decision-making combining with grey relations" to deal with real case of effective dwelling house walls with attributes values determined at intervals.

In fact, the previously proposed algorithms for MCDM adopt the "additive" algorithm after parameterization and normalization of the evaluation indexes till now, and some algorithms even contain artificial and subjective factors, such as VIKOR (Jahan et al., 2011), TOPSIS and MOORA (Moradian et al., 2019; Athawale et al., 2011). From the perspective of "simultaneous optimization of multiple indexes", the previous algorithms have the inherent shortcomings of "additive", which is equivalent to taking the form of "union" in probability theory (Brémaud, 2020) and far from the nature of "simultaneous optimization of multiple indexes". In fact, from the point of view of probability theory, "simultaneous optimization of multiple indexes" should be more appropriate to take the form of "joint probability" for the "simultaneous optimization of multiple indexes" (Brémaud, 2020). In addition, since the introduction of subjective factors, the relevant approaches are at most a semi – quantitative methods in some sense.

In this paper, a quantitative method for MCDM assessment is developed on basis of probability theory, a new concept of favorable probability is introduced, and the total favorable probability of an alternative is the decisive parameter for the alternative to win in the optimal process comparatively.

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1. New approach for multi-criteria decision-making

1.1. Concept of favorable probability for multi-criteria decision-making

In general, every alternative in the multi-criteria decision-making could present many characteristics in different aspects; each performance index of an alternative reflects partial feature of its full characteristics in some sense. Some performance indexes might be beneficial (welcomed) to the decision-making, but other performance indexes might be unbeneficial to the decision-making. An actual alternative is the integral body of both beneficial and unbeneficial performance indexes to the decision-making. It is not possible for an alternative to have only full beneficial or unbeneficial performance index to the decision-making. So, all the performance indexes can be divided into beneficial and unbeneficial types to the decision-making naturally.

A typical example is the design and manufacture of an spaceship, the strength and ductility of a material are the beneficial material performance indexes for the material selection, while the specific gravity (density) is unbeneficial material performance index to the material optimization.

In general, the beneficial performance indexes have the property values of the higher the better, and unbeneficial performance indexes have the properties of the lower the better to the decision-making.

In the viewpoint of impersonal analysis, an overall consideration for alternative for decision-making is needed, which makes the multi-criteria decision-making a comprehensive and systemic task. Therefore, both the beneficial or unbeneficial performance indexes should be dealt with appropriately, so as to propose an overall consideration to the multi-criteria decision-making quantitatively.

1.2. Probability based quantitative approach of multi-criteria decision-making

As a quantitative assessment to the term "the higher the better" for a performance index of a candidate, a new concept of favorable probability can be introduced; the favorable degree of the performance index in the decision-making competition comparatively can be quantitatively represented by the favorable probability.

From the principle of simplicity, it assumes that the partial favorable probability of a performance index with the character of "the higher the better" (beneficial performance index) in the decision-making process is positively correlative to this performance index linearly, i.e.:

$$P_{ij} \propto X_{ij},\ P_{ij} = \alpha_j X_{ij},\ i = 1, 2,..., n,\ j = 1, 2,..., m. \qquad (1)$$

In Eq. (1), X_{ij} represents the *j-th* performance index of the *i-th* alternative; P_{ij} is the partial favorable probability of the beneficial performance index X_{ij}; n is the total number of alternatives in the evaluation group involved; m the total number of performance index of each alternative in the group; α_j is the normalized factor of the *j-th* performance index of the beneficial typed performance.

Furthermore, according to the general principle of probability theory (Brémaud, 2020), the summation of each P_{ij} for the index i in j-th performance index is normalized and equal to 1, i.e., $\sum_{i=1}^{n} P_{ij} = 1$, thus, it obtains:

$$\sum_{i=1}^{n} \alpha_i X_{ij} = \sum_{i=1}^{n} P_{ij} = 1,\ \alpha_j = 1/(n\overline{X_j}). \qquad (2)$$

$\overline{X}_j$ is the average value of the *j-th* performance index in the alternative group involved.

Similarly, partial favorable probability of the unbeneficial performance index X_{ij} to the candidate is negatively correlative to its performance index linearly, i.e.:

$$P_{ij} \propto (X_{j\max} + X_{j\min} - X_{ij}),\ P_{ij} = \beta_j(X_{j\max} + X_{j\min} - X_{ij}),$$
$$i = 1, 2,..., n,\ j = 1, 2,..., m. \qquad (3)$$

In Eq. (3), $X_{j\max}$ and $X_{j\min}$ present the maximum and minimum values of the performance index X_j in the alternative group, respectively; β_j is the normalized factor of the *j-th* performance index of the unbeneficial typed performance.

Correspondingly, by using the general principle of probability theory (Brémaud, 2020), it obtains:

$$\beta_j = 1/[n(X_{j\max} + X_{j\min}) - n\overline{X_j}]. \qquad (4)$$

Furthermore, according to basic probability theory (Brémaud, 2020), the total / comprehensive favorable probability of the *i-th* alternative to be selected is the product of its partial favorable probability of each performance index P_{ij}, i.e.:

$$P_i = P_{i1} \cdot P_{i2} \cdots P_{im} = \prod_{j=1}^{m} P_{ij}. \qquad (5)$$

If there is weight q_{ij} for the j attribute in the i alternative of a solution, the total / comprehensive favorable probability of the *i-th* alternative is the weighted product of its partial favorable probability of each performance index P_{ij} in such a case, i.e.:

$$P_i = P_{i1}{}^{q_{i1}} \cdot P_{i2}{}^{q_{i2}} \cdots P_{im}{}^{q_{im}} = \prod_{j=1}^{m} P_{ij}{}^{q_{ij}}. \qquad (6)$$

Finally, the total favorable probability of an alternative is the decisive parameter in the MCDM to win the competition comparatively, the winner / victor is with the maximum total favorable probability.

By far, the concept of favorable probability for MCDM and its evaluation are developed.

2. Application of the new multi-criteria decision-making approach

The new approach is applied to deal with the following multi-criteria decision-making problems.

2.1. Selection for effective dwelling house walls

Zavadskas et al. (Zavadskas et al., 2008a) once developed "specific method of multiple criteria decision-making combining with grey relations" to deal with real case of attributes values determined at intervals. Table 1 shows the initial decision-making index for effective dwelling house walls with values expressed in intervals, which is cited from Zavadskas et al. (2008a).

In Table 1, b_{ij} means the biggest value – upper limit of the j attribute in the i alternative of a solution, w_{ij} means the lowest value – lower limit of the j attribute in the i alternative of a solution; "max" means "the higher the better", i.e., favorable performance index, "min" means "the lower the better", i.e., unbeneficial performance index in the MCDM evaluation.

Furthermore, we could take the arithmetic mean value of each value in Table 1 as the evaluation index, which is presented in Table 2. Table 3 shows evaluated results for partial favorable probability and total favorable probability, as well as ranking according to the total favorable probability of each alternative.

The rank in Zavadskas et al. (2008a) is A1 > A3 > A2 > A4, which is the same as our rank in Table 3.

2.2. Multi-criteria selection of project managers

Beside, Zavadskas et al. (Zavadskas et al., 2008b) applied the "specific method of multiple criteria decision-making combining with grey relations" to deal with project manager problem.

Table 4 shows the initial decision-making index for project manager problem, which is cited from Zavadskas et al. (2008b).

In Table 4, $\overline{x_{ij}}$ means the biggest value – upper limit of the j attribute in the i alternative of a solution, x_{ij} means the lowest value – lower limit of the j attribute in the i alternative of a solution; "max" means "the higher the better", i.e., favorable performance index, "min" means "the lower the better", i.e., unbeneficial performance index in the MCDM evaluation of the project manager selection.

Again, let's take the arithmetic mean value of each value in Table 4 as the evaluation index, which is presented in Table 5. Table 6 shows evaluated results for partial favorable probability and total favorable probability, as well as ranking according to the total favorable probability of each alternative.

The rank in Zavadskas et al. (2008b) for the project manager selection is project manager 2 > project manager 3 > project manager 1, which is the same as our rank in Table 6.

2.3. Multi-criteria selection of contractor for construction works

In addition, Zavadskas et al. (2010) studied contractor selection problem for construction works by applying SAW-G and topsis grey techniques.

Table 7 shows the initial decision-making index values of contractor selection for construction works problem, which is cited from Zavadskas et al. (2010).

Once more, we take the arithmetic mean value of each value in Table 4 as the evaluation index, which is presented in Table 8. Table 9 shows evaluated results for partial favorable probability and total favorable probability, as well as ranking according to the total favorable probability of each alternative.

The rank of the contractor selection for construction works problem in Zavadskas et al. (2010) is A1 > A2 > A3 > A5 > A4, which is the same as our rank in Table 9.

The applications of the new approach in above cases indicate its validity.

Table 1. Initial decision-making matrix with values expressed in intervals

Alternate No.	Durability		Thermal transmit.		Cost		Weight of wall		Human expenditure	
Weight q	0.21		0.33		0.26		0.09		0.11	
Index	X1		X2		X3		X4		X5	
Optimum	max		min		min		min		min	
Alternative	w1	bi	w2	b2	w3	b3	w4	b4	w5	b5
A1	75	100	0.22	0.25	72.08	94.71	590	652	4.60	4.60
A2	75	100	0.22	0.25	89.01	100.93	596	625	4.60	4.60
A3	75	100	0.21	0.25	80.32	96.42	581	604	4.60	4.60
A4	25	25	0.24	0.27	67.76	98.10	455	479	4.55	5.01

Table 2. Arithmetic mean value of decision-making index

Index	X1	X2	X3	X4	X5
Attribute weight q	0.21	0.33	0.26	0.09	0.11
1	87.5	0.235	83.395	621	4.6
2	87.5	0.235	94.97	610.5	4.6
3	87.5	0.23	88.37	592.5	4.6
4	25	0.255	82.93	467	4.78

Table 3. Evaluated results for partial favorable probability, total favorable probability and ranking

Probability	P_{X1}	P_{X2}	P_{X3}	P_{X4}	P_{X5}	P_t	Rank
1	0.3043	0.2538	0.2611	0.2266	0.2524	0.2628	1
2	0.3043	0.2538	0.2291	0.2317	0.2524	0.2545	3
3	0.3043	0.2589	0.2474	0.2405	0.2524	0.2622	2
4	0.0870	0.2335	0.2624	0.3013	0.2429	0.2010	4

Table 4. Initial decision-making index with the criterion values described in intervals and normalized weighted matrix

Index	X_1		X_2		X_3		X_4		X_5		X_6	
Optimum	max		max		max		max		max		min	
Weight *q*	0.25		0.15		0.12		0.20		0.13		0.15	
Maneger	$\underline{x_1}$	$\overline{x_1}$	$\underline{x_2}$	$\overline{x_2}$	$\underline{x_3}$	$\overline{x_3}$	$\underline{x_4}$	$\overline{x_4}$	$\underline{x_5}$	$\overline{x_5}$	$\underline{x_6}$	$\overline{x_{i6}}$
1	50	60	40	55	10	20	50	70	50	45	30	40
2	70	80	60	70	40	45	60	75	70	80	70	60
3	60	70	55	70	30	40	70	80	55	65	40	50

Table 5. Arithmetic mean value of decision-making index for project manager selection

Index	X_1	X_2	X_3	X_4	X_5	X_6
Weight factor *q*	0.25	0.15	0.12	0.20	0.13	0.15
1	55	47.5	15	60	47.5	35
2	75	65	42.5	67.5	75	65
3	65	62.5	35	75	60	45

Table 6. Evaluated results for partial favorable probability, total favorable probability and ranking for project manager selection

Probability	P_{X1}	P_{X2}	P_{X3}	P_{X4}	P_{X5}	P_{X6}	P_t	Rank
1	0.2821	0.2714	0.1622	0.2963	0.2603	0.4194	0.2783	3
2	0.3846	0.3714	0.4595	0.3333	0.4110	0.2258	0.3537	1
3	0.3333	0.3571	0.3784	0.3704	0.3288	0.3548	0.3519	2

Table 7. Initial decision-making index values of contractor selection for construction works problem

Index	X_1		X_2		X_3		X_4		X_5		X_6	
Optimum	max		max		max		min		max		max	
Alternative	w1	bi	w2	b2	w3	b3	w4	b4	w5	b5	w6	b6
A1	11	15	10	15	3.30	4.5	35	48	0.152	0.203	1	2
A2	10	14	7	13	2.54	3.68	40	58	0.111	0.162	1	2
A3	14	18	5	9	1.95	2.46	42	53	0.079	0.121	1	3
A4	12	16	1	4	0.42	1.73	15	63	0.01	0.054	1	2
A5	6	10	2	9	0.62	2.67	10	46	0.012	0.122	1	2

Table 8. Arithmetic mean value of decision-making index of contractor selection for construction works problem

Index	X_1	X_2	X_3	X_4	X_5	X_6
1	13	12.5	3.9	41.5	0.1775	1.5
2	12	10	3.11	49	0.1875	1.5
3	16	7	2.205	47.5	0.1	2
4	14	2.5	1.075	39	0.0275	1.5
5	8	5.5	1.645	28	0.067	1.5

Table 9. Evaluated results for partial favorable probability, total favorable probability and ranking of contractor selection for construction works problem

Probability	P_{X1}	P_{X2}	P_{X3}	P_{X4}	P_{X5}	P_{X6}	$P_t \times 10^5$	Rank
1	0.2063	0.3333	0.3268	0.1972	0.3172	0.1875	26.3681	1
2	0.1905	0.2667	0.2606	0.1556	0.3351	0.1875	12.9371	2
3	0.2540	0.1867	0.1848	0.1639	0.1787	0.2500	6.4139	3
4	0.2222	0.0667	0.0901	0.2111	0.0492	0.1875	0.2596	5
6	0.1270	0.1467	0.1378	0.2722	0.1197	0.1875	1.5690	4

Conclusions

From above discussion, the new approach for multi-criteria decision-making developed on basis of probability theory is an appropriate method, which takes all the possible performance indexes into overall consideration comprehensively by the introduction of favorable probability. All the performance indexes are divided into beneficial and unbeneficial performance index types, which contribute to the partial favorable probability of the alternative in positively correlative or negatively correlative manners, respectively. The total favorable probability of an alternative is the product of its partial favorable probability of each performance index, which decides the final result of the multi-criteria decision-making definitely and comprehensively.

Declarations

Authors declared that there is no conflict of interest.

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