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ASSESSMENT OF RECENT GOCE-BASED GLOBAL GEOPOTENTIAL MODELS AND EGM2008 IN NIGER REPUBLIC

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Abstract. In this study, we assessed recent GOCE-based Global Geopotential Models (GGMs) and EGM2008 in Niger. The combined GGMs EIGEN_6C4, GECO and EGM2008 were evaluated up to their maximum degree and order (d/o) 2,190 to select the one for gravity database densification. The following pure satellite GGMs were assessed for the modelling of the long and medium wavelengths in geoid computation: GGM05G, ITU_GGC16, EIGEN_6S4v2 and the fifth releases from direct (DIR5), space-wise (SPW5) and time-wise (TIM5) approaches. The GGMs are compared to terrestrial gravity data and geoid heights from GNSS/Levelling points before and after applying spectral enhancement method (SEM) by residual terrain model (RTM) for combined models and by RTM and the coefficients of selected combined GGM for pure satellite models. The agreements of combined GGMs with terrestrial gravity data and GNSS/Levelling points, in terms of root mean square (RMS) are about 4.88 to 5.02 mGal and 0.14 to 0.16 m, respectively. EIGEN_6C4 was selected as it showed the best performance in terms of geoid height differences and the probability of 3-sigma rule for gravity anomaly differences. At d/o 200, DIR5 showed a good agreement with terrestrial gravity data (5.04 mGal) and GNSS/Levelling points (0.15 m) after applying SEM, it was then retained. All GOCE-based models exhibited a good performance in long and medium wavelengths confirming the good recovery of the gravity field by the spatial gravity mission in these spectral bands.

Keywords: geopotential model, GOCE, EGM2008, geoid, gravity anomaly, GNSS/Levelling, Niger.

Introduction

The Global Geopotential Model (GGM) is a set of coefficients of a spherical harmonic expansion to a degree and order (d/o) of the Earth gravity field for global analysis. There are two categories of GGMs: pure satellite model and combined model which is obtained by merging spatial gravity measurements and terrestrial data eg. (Förste et al., 2016; Förste et al., 2014; Gatti & Reguzzoni, 2017). The last three satellite gravity missions are CHAllenging Mini-satellite Payload (CHAMP), Gravity Recovery and Climate Experiment (GRACE) and Gravity Field and Steady State Ocean Circulation Explorer (GOCE). Numerous GGMs were derived from the observations that represent the final scientific output for applications in oceanography, solid earth physics and geodesy. The GGMs are freely available on the website of the International Centre for Global Earth Models (ICGEM) (Barthelmes & Köhler, 2016).

The expected accuracies from GOCE are of 1 mGal and 1–2 cm, respectively, for gravity anomalies and geoid, at spatial resolution of approximately 100 km half wavelength or d/o 200 of spherical harmonics (Drinkwater, Floberghagen, Haagmans, Muzi, & Popescu, 2003). Up to date, five releases of GOCE-based GGMs have been developed, based on observation data in different time spans, by ESA's (European Space Agency) High Level Processing Facility (HPF). Measured data are processed through three different approaches: direct, time-wise and space-wise (Pail et al., 2011). The 5th releases were based on the complete mission lifetime including the lower orbit data up to the re-entry of the satellite in November 2013 (Tocho & Vergos, 2015). Besides the ESA's solutions, some models are based on the combination with other data sources, such as terrestrial gravity, satellite altimetry, or with CHAMP and GRACE eg. (Förste et al., 2016; Förste et al., 2014). GECO is an update of EGM2008 with GOCE data (Gilardoni, Reguzzoni, & Sampietro, 2015).

In the remove-compute-restore (RCR) technique, the contribution of long and medium wavelengths of the

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gravity field and the geoid, is computed from a pure satellite GGM (Omang & Forsberg, 2000). The accuracy of the achieved geoid depends on that of the used GGM and it is sensitive to d/o ranging from 150 and 250 (Varga, Grgic, Bjelotomić, & Bašić, 2018). The enhanced high-resolution combined GGMs by the residual terrain model (RTM) are also used for gravity database densification in areas with sparse data (Ulotu, 2009).

The gravity field quantities computed from a GGM are subject to the signal omission error comprising high frequency signals that cannot be represented by a truncated spherical harmonic series expansion (Hirt, Featherstone, & Marti, 2010). In order to select the best GGM for gravity field modelling and geoid computation by RCR, several validation studies were carried out around the world, at local, regional or global scales eg. (Apeh, Moka, & Uzodinma, 2018; Foroughi, Afrasteh, Ramouz, & Safari, 2017; Godah & Krynski, 2015; Yakubu, Ferreira, & Asante, 2017). The quantities derived from GGMs are compared to terrestrial gravity anomalies, deflection of the vertical, existing geoids, GNSS/Levelling data and those from other GGMs. Some studies are based on direct comparison by simple differences between GGMs and terrestrial data at different d/o of truncation (Yakubu et al., 2017). Statistical measures such as standard deviation (σ) and root mean square (RMS) of differences are analysed as function of d/o of truncation. Due to different spectral content between GGMs and terrestrial data, low-pass filters are commonly used to make the data comparable, such as Gaussian, average and inverse distance filters (Godah & Krynski, 2015; Yakubu et al., 2017). However, the low-pass filters are more suitable for densely distributed or gridded data. The spectral enhancement is also used, the GGMs are completed with spherical harmonic coefficients of high-resolution combined GGMs such as EGM2008, and RTM (Hirt, Gruber, & Featherstone, 2011). The RTM technique is capable of modelling major parts of high-resolution GGM omission error and it can improve geoid determination in mountainous area (Hirt et al., 2010). The global assessment results are available on ICGEM website (Barthelmes & Köhler, 2016). Several validation studies showed high performances and improvements worldwide of GOCE-based GGMs in medium and long wavelengths (El Brirchi & El Azzab, 2011; Godah & Krynski, 2015; Hirt et al., 2011; Yakubu et al., 2017). Also, the latest releases of GOCE-based GGMs derived from large time span measurements showed better performances (Tocho & Vergos, 2015).

Up to date, there is no dedicated studies to assess the GGMs in general and GOCE-based ones in Niger. Some studies were done on the entire African continent and neighbouring countries such as Algeria, without a particular emphasis on Niger (Abd-Elmotaal, 2015; Benahmed Daho, 2010; Merry, 2009). This could be due to the inaccessibility or unavailability of sufficient validation data, including GNSS/Levelling points. The Niger Republic does not yet have a gravimetric geoid model and the gravity

data are not densely distributed. The efforts are ongoing to compute the first geoid model in Niger. After the assessment of digital elevation models, the most suitable was used to compute high-resolution RTM effects for gravity field modelling and geoid computation (Ibrahim Yahaya & El Azzab, 2018, 2019).

The main objective of this study is to select the best pure satellite GGM for the computation of long and medium wavelengths contributions in geoid determination by RCR and the combined model for gravity database densification in Niger. We therefore assess the recent GOCE-based GGMs and EGM2008. The pure satellite GGMs under evaluation are the fifth releases GO_CONS_GCF_2_SPW_R5 (SPW5) (Gatti & Reguzzoni, 2017), GO_CONS_GCF_2_TIM_R5 (TIM5) (Brockmann et al., 2014) and GO_CONS_GCF_2_DIR_R5 (DIR5) (Bruinsma et al., 2013), we included ITU_GGC16 (Akyilmaz et al., 2016), EIGEN-6S4v2 (Förste et al., 2016) and GGM05G (Bettadpur et al., 2015). The combined GGMs are EGM2008 (Pavlis, Holmes, Kenyon, & Factor, 2012), EIGEN-6C4 (Förste et al., 2014) and GECO (Gilardoni et al., 2015). The GGMs are compared to terrestrial gravity data and GNSS/Levelling points before and after applying the spectral enhancement method by RTM, and by RTM and coefficients of combined GGMs, respectively, to combined GGMs and pure satellite ones. The differences are analysed by statistical measures.

1. Materials and methods

1.1. Theory

The geoid undulation N_{GGM} and the gravity anomalies Δg_{GGM} from a GGM are computed, respectively, using the expressions (Hofmann-Wellenhof & Moritz, 2005):

$$N_{GGM} = \frac{GM}{r\gamma_0} \times$$

$$\sum_{n=2}^{+\infty} \left(\frac{R}{r}\right)^n \sum_{m=0}^{n} \left(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}\left(\cos\theta\right) \quad (1)$$

and

$$\Delta g_{GGM} = \frac{GM}{r^2} \times$$

$$\sum_{n=2}^{+\infty} (n-1)\left(\frac{R}{r}\right)^n \sum_{m=0}^{n} \left(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}\left(\cos\theta\right)$$

$$(2)$$

with γ_0 the normal gravity on the reference ellipsoid, r, λ, θ geocentric radius, longitude and colatitude of computation point, respectively. R is the reference radius, GM is the product of gravitational constant and the mass of the Earth, n, m are the degree and order of spherical harmonic, $\overline{P}_{nm}$ are the fully normalised Legendre functions, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are fully normalised Stokes' coefficients of the disturbing potential. The infinite series is usually

truncated at the maximum resolvable degree n_{max} that is associated with a shortest resolvable wavelength at Earth's surface. In this study, we consider the terminology tailored in Šprlák, Gerlach, and Pettersen (2012): d/o 2 to 100 for low frequencies, 101 to 250 for medium wavelengths, 251 to 2,190 for high frequencies and the very-high frequencies for d/o above 2,190.

1.2. Spectral enhancement method

The main idea of the spectral enhancement method (SEM) is to fill the spectral gap between GGMs and terrestrial data (Hirt et al., 2011). For high-resolution GGMs such as EGM2008, omission error estimates sourced from RTM are used to recover the spectral band beyond n_{max} = 2,190. A GGM (e.g. pure satellite model) under evaluation is evaluated up to degree k_1 and the spectral bands beyond this degree (from k_1+1) are recovered using a high-resolution GGM and RTM omission error estimates. The omission error estimates are limited to the resolution of the digital elevation model used for RTM computation, for 3 arc-seconds resolution, it corresponds to d/o 216,000 (Hirt et al., 2011). The SEM is used for recovering various gravity field quantities, such as gravity anomalies, geoid undulations, height anomalies and deflections of the vertical (Hirt et al., 2010). The simplified principle of SEM is illustrated in Figure 1.

Figure 1. Simplified principle of Spectral Enhancement Method (SEM) (Hirt et al., 2011)

1.3. Study area and terrestrial data

The study area is located between 11° to 24° North and 0° to 16° East, and covers the Niger Republic and few other countries in the Central West Africa. The terrestrial gravity data cover the study area, they are provided by the International Gravimetric Bureau (BGI). We also retrieved gravity stations from ORSTOM (*Office de Recherche Scientifique et Technique Outre Mer*-France) technical report (Rechenmann, 1966). The quoted observation errors of simple Bouguer anomalies in Niger gravity database for the most unfavourable cases are 3.4 mgal and 5.4 mgal below and above the 16th parallel, respectively. The merged data are 9,737 points of free air gravity anomalies Δg_{FA}, the values refer to GRS80 (Geodetic Reference System 1980) (Ibrahim Yahaya, El Brirchi, & El Azzab, 2017b). Figure 2 presents the spatial distribution of gravity stations in the study area. The gravity database contains void areas in the southern of Algeria and Libya, and in the northern of Chad, Mali and Nigeria.

1.4. GNSS/Levelling data

The available 39 GNSS/Levelling points are located in the southwest part of Niger (Ibrahim Yahaya, El Brirchi, & El Azzab, 2017a). They belong to the second order and the complementary levelling networks, with the accuracies of 5√S mm and 5√S cm, respectively, where S is the levelling distance in km. They are also measured by differential GNSS using the static mode. We also retrieved 11 points of Nigerian geodetic network that are located in our study area from (Orupabo, Opuaji, & Adekunle, 2014). The GNSS/Levelling points are plotted on Figure 3. The geoid undulation N_{GPS} from GNSS/Levelling point is the difference between the ellipsoid height h and the orthometric height H:
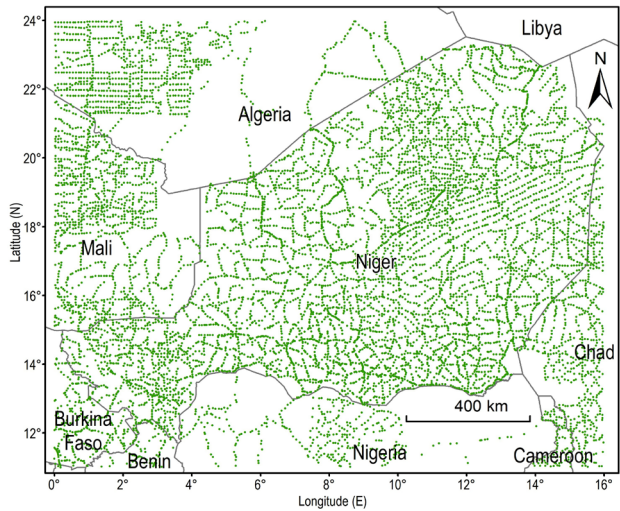
$$N_{GPS} = h - H .\qquad(3)$$

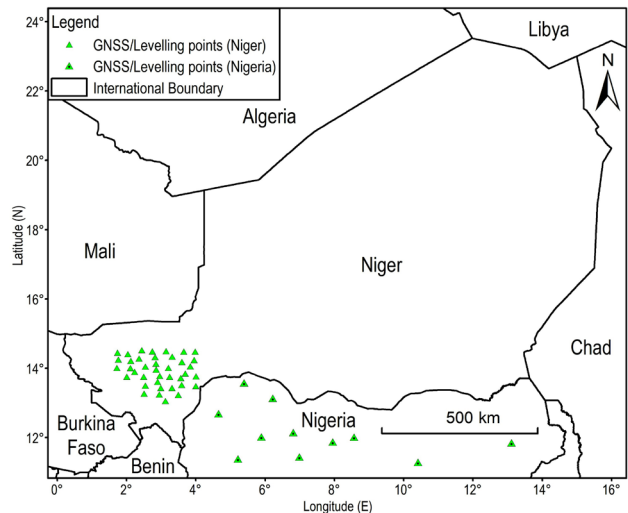Figure 2. Spatial distribution of terrestrial gravity data

Figure 3. Spatial distribution of GNSS/Levelling points

1.5. Characteristics of GGMs under evaluation and computation of gravimetric quantities

The GGMs were downloaded from the website of the ICGEM (Barthelmes & Köhler, 2016), Table 1 presents their characteristics. The space-wise SPW5 and time-wise TIM5 solutions are GOCE-only GGMs, the direct approach solution DIR5 combines GRACE and LAGEOS data.

We computed gravity anomalies and geoid heights from all GGMs for d/o from 100 to 300 with five degrees of spectral interval. For gravity anomalies, 1.5' spaced grids were computed using the calculation service of ICGEM and values were extracted at each gravity station. The geoid undulations are computed at each GNSS/Levelling point using GRAFIM Software (Janák & Šprlák, 2006). The combined models are also evaluated up to their maximum d/o, 2,190. All quantities refer to GRS80.

Table 1. Models under evaluation
S = Satellite, G = Gravity, A = Altimetry

Model	Year	d/o	Data source	Reference
SPW5	2017	330	S (GOCE)	(Gatti & Reguzzoni, 2017)
ITU_GGC16	2016	280	S (GRACE, GOCE)	(Akyilmaz et al., 2016)
EIGEN-6S4v2	2016	300	S (GOCE, GRACE, LAGEOS)	(Förste et al., 2016)
GECO	2015	2190	S(GOCE), EGM2008	(Gilardoni et al., 2015)
GGM05G	2015	240	S (GRACE, GOCE)	(Bettadpur et al., 2015)
EIGEN-6C4	2014	2190	S(GOCE, GRACE, LAGEOS), G, A	(Förste et al., 2014)
TIM5	2014	280	S (GOCE)	(Brockmann et al., 2014)
DIR5	2014	300	S (GOCE, GRACE, LAGEOS)	(Bruinsma et al., 2013)
EGM2008	2008	2190	S(GRACE), G, A	(Pavlis et al., 2012)

1.6. Assessment of combined models

The quantities X_{Comb}^{2190} derived from combined GGMs up to their maximum d/o 2,190 are compared to those of terrestrial data and GNSS/Levelling points X before and after applying SEM. Before the SEM, the difference δX is given by:

$$\delta X = X - X_{Comb}^{2190}, \qquad (4)$$

where X stands for gravity anomalies Δg or geoid undulations N.

We used the grids of RTM effects derived from the reference topographic surface corresponding to about 9 km of spatial scale or d/o beyond 2,190 (Ibrahim Yahaya &

El Azzab, 2018). The values of the indirect effect on geoid and the RTM gravity effect were extracted at each gravity station and GNSS/Levelling point. By applying the SEM using the RTM contributions X_{RTM} to gravity anomalies or geoid, the differences are given by:

$$\delta X = X - (X_{Comb}^{2190} + X_{RTM}). \qquad (5)$$

The descriptive statistics of the differences are analysed in terms of maximum (Max), minimum (Min), mean (μ), standard deviation (σ), root mean square (RMS) and histograms. We performed the Jarque-Bera normality tests at 5% and 10% of confidence levels on gravity anomaly differences. According to the 3-sigma rule in statistics, 99.7% of the data values fall within three σ of μ in either direction for normal distributions: $P[|\delta\Delta g - \mu| \leq 3\sigma] = 99.7\%$. We computed the probability for all GGMs. The best combined GGM will be used in the SEM for the pure satellite GGMs.

1.7. Assessment of pure satellite GGMs

The gravity anomalies or the geoid heights X_{GGM}^{k} derived from pure satellite GGMs up to d/o $k = 1, 105, 110…300$ are compared to terrestrial free air anomalies and geometric geoid heights respectively. The differences are given by:

$$\delta X = X - X_{GGM}^{k}. \qquad (6)$$

By applying the SEM, the differences are computed as follows:

$$\delta X = X - \left(X_{GGM}^{k} + X_{Comb}^{k+1\,to\,2190} + X_{RTM} \right), \qquad (7)$$

where $X_{Comb}^{k+1\,to\,2190}$ is the modelled quantity by the combined model from d/o $k+1$ to 2,190. The RMS values of the differences are analysed as function of d/o k.

2. Results and discussions

2.1. Comparison of combined models

2.1.1. Gravity anomalies

The histograms of gravity anomaly differences are plotted in Figure 4 for all combined GGMs before and after applying SEM, they are close to normal distributions from the same μ and σ represented by the solid red lines. The Jarque-Bera normality tests showed that the differences of gravity anomaly do not follow normal distributions. The 3-σ rule confirms this fact for all GGMs, the probabilities are less than 99% (Table 2). The distribution of gravity anomalies is not normal in general, and in our case, the existence of outliers in validation data could also explain the finding.

The statistics of the differences between the measured free-air anomalies and those modelled from the combined models, before and after SEM, are presented in Table 2. At 0.1 mGal order, the models are similar before and after the application of SEM. EGM2008 has the smallest RMS, 4.88 mGal, and GECO presents the greatest values, 5.02 mGal. The models show a slight increase in the range of 0.01 to

Table 2. Statistics of differences between terrestrial gravity data and combined GGMs at d/o 2190 (in mGal)

	Model	MIN	MAX	μ	σ	RMS	$P[\|\delta\Delta g - \mu\| \leq 3\sigma]$	No.
Gravity anomalies	Δg_{FA}	−50.84	108.22	−0.65	15.87	15.89	–	
Differences before SEM	EGM2008	−36.70	45.90	−1.28	4.71	4.88	98.57%	
	EIGEN-6c4	−37.15	48.01	−1.32	4.78	4.96	98.77%	
	GECO	−36.71	48.48	−1.31	4.85	5.02	98.74%	
Differences after SEM	Δg_{RTM}	−12.34	24.65	−0.20	1.13	1.14	–	9737
	EGM2008	−42.65	45.69	−1.08	4.77	4.89	98.73%	
	EIGEN-6c4	−41.60	47.79	−1.12	4.85	4.98	98.80%	
	GECO	−41.48	48.26	−1.12	4.91	5.03	98.77%	

0.02 mGal for RMS values after applying SEM. There is no improvement according to RMS values, other authors such as Šprlák, Gerlach, and Pettersen (2015) reported the contrary. On the other hand, we notice a decrease of 0.2 mGal for the mean values of differences after the application of SEM, thus the reduction of the systematic bias. Also the probabilities $P[\|\delta\Delta g - \mu\| \leq 3\sigma]$ increase from 98.57% to 98.73%, 98.77% to 98.80% and 98.74% to 98.77%, respectively, for EGM2008, EIGEN_6C4 and GECO. EIGEN_6C4 showed the greatest probability values before and after SEM.

2.1.2. Comparison of geoid heights

The statistics of the difference between the geoid heights from GNSS/Levelling points and those modelled from the GGMs in Niger and northern Nigeria are presented in Table 3.

The first finding is that there is no improvement in geoid height in terms of σ and RMS by RTM as it was reported in Šprlák et al. (2015), contrary to the results of other studies (Hirt et al., 2010; Kadlec, 2011). The RTM contributions to geoid heights are very low (less than 1 cm) at the GNSS/Levelling points in our study area. EIGEN-6c4 and GECO have almost the same σ and RMS values of differences. In Niger, they are slightly better than EGM2008, while in northern Nigeria EGM2008 performs better.

According to the results obtained with the gravity anomalies, EGM2008 is the candidate GGM for spectral enhancement and densification of the gravity database but it does not contain GOCE data. However, better precision is expected from the other two models in long and medium wavelengths because they contain GOCE data and EIGEN-6C4 showed the greatest probability according to
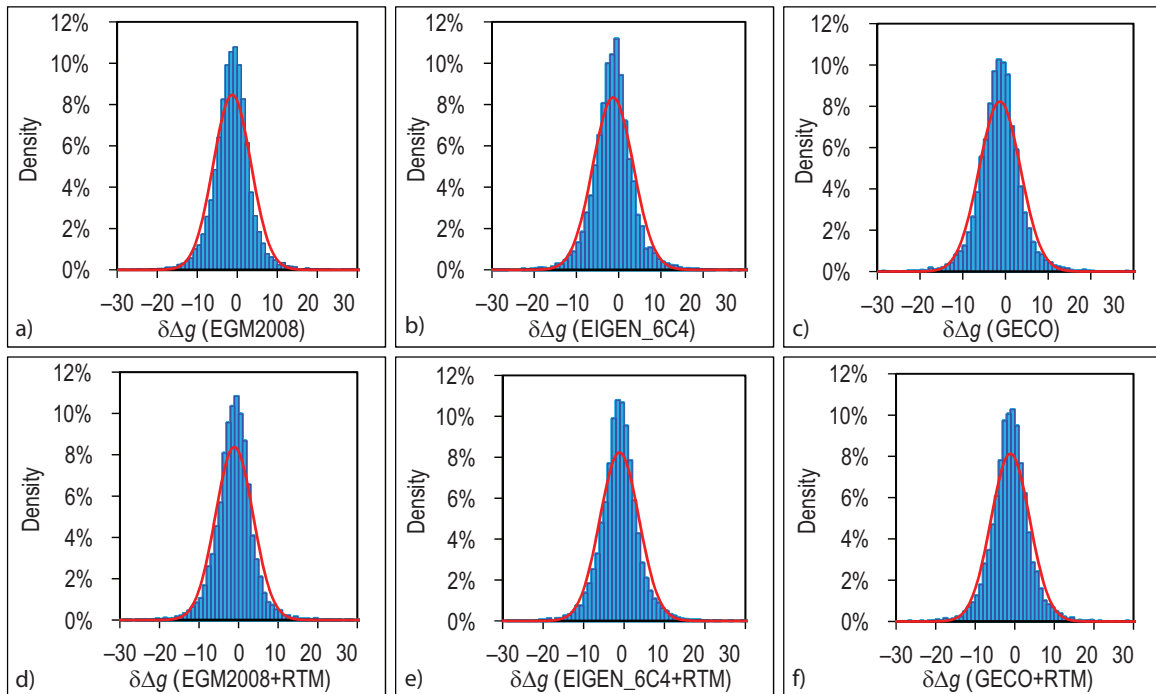

Figure 4. Histograms of gravity anomaly differences for combined GGMs up to d/o 2,190 and normal distributions from the same μ and σ (Red). Before SEM: a – EGM2008; b – EIGEN_6C4; c – GECO. After SEM: d – EGM2008; e – EIGEN_6C4; f – GECO

Table 3. Statistics of differences between GNSS/Levelling data and combined models at d/o 2190 (in metres)

Area	SEM	Model	MIN	MAX	μ	σ	RMS	No.
Niger	Before	EGM2008	−0.27	0.44	0.04	0.15	0.16	39
		EIGEN-6c4	−0.23	0.43	0.02	0.14	0.14	
		GECO	−0.25	0.44	0.01	0.14	0.14	
	After	N_{RTM}	−0.003	0.005	0.001	0.002	0.002	
		EGM2008	−0.27	0.44	0.04	0.15	0.16	
		EIGEN-6c4	−0.23	0.43	0.02	0.14	0.14	
		GECO	−0.25	0.44	0.02	0.14	0.14	
Nigeria	Before	EGM2008	−1.05	1.95	0.46	0.83	0.92	11
		EIGEN-6c4	−1.10	1.89	0.47	0.85	0.94	
		GECO	−1.10	1.88	0.48	0.86	0.95	
	After	N_{RTM}	−0.007	0.001	−0.002	0.002	0.003	
		EGM2008	−1.06	1.95	0.46	0.83	0.92	
		EIGEN-6c4	−1.10	1.89	0.47	0.85	0.94	
		GECO	−1.10	1.88	0.47	0.86	0.95	

3-sigma rule. Also, for global assessments EIGEN-6C4 is presently used as a reference model (Barthelmes & Köhler, 2016). We therefore retained EIGEN-6C4. In order to have an overview of the performance of these combined models in long and medium wavelengths, we included them in the comparison of "pure satellite" GGMs in the following section.

2.2. Comparison of pure satellite GGMs with gravity anomalies

2.2.1. Before SEM

The RMS values of differences for gravity anomalies as a function of truncation d/o prior to SEM are shown in Figure 5a. All models showed similar performance from d/o 100 to 215 compared to terrestrial data, RMS values decreased gradually with high d/o. From d/o 215, GGM05G stands out from other GGMs, with the largest gap up to its maximum degree, 240. Similarities between models continue for other GGMs up to d/o 230 where ITU_GGC16 and TIM5 stand out from other models.

From d/o 240, all pure satellite models become stable. ITU_GGC16 and TIM5 perform similarly up to d/o 280, also for EIGEN_6S4v2 and DIR5 up to d/o 300, and SPW5 is more accurate up to this d/o. Beyond d/o 200, the RMS values of differences for the combined models decrease progressively and similarly until d/o 300, GECO is less accurate than EGM2008 and EIGEN-6c4. EGM2008 appears similar to EIGEIN-6C4 and is better than other models. In these spectral bands, the combined models are generally better than the pure satellite GGMs due to the integration of terrestrial data that contain the full gravity spectrum.

2.2.2. After SEM

The RMS values for gravity anomalies differences as function of d/o after applying SEM are shown in Figure 5b. Up to d/o 180, all models have RMS curves coinciding with that of EIGEN-6C4 up to d/o 2,190 and enhanced by RTM, except GGM05G and EGM2008, which show poor performance compared to other models. The result of EGM2008 is justified by the fact that it does not contain GOCE data, on the other hand GGM05G combines
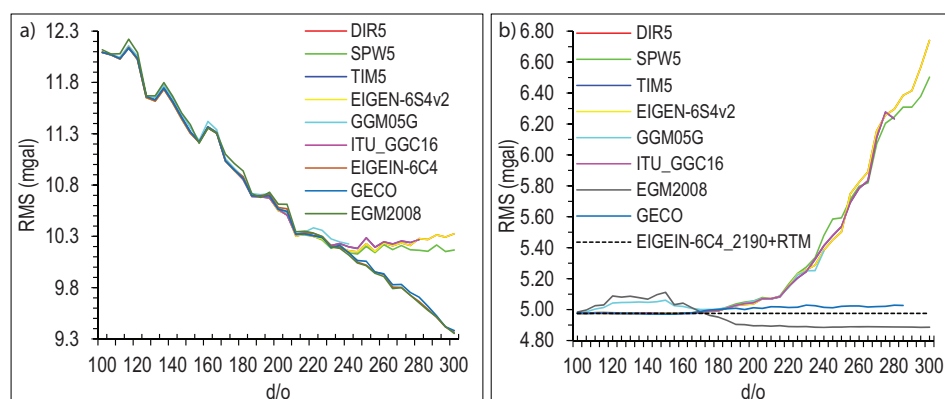
Figure 5. RMS of gravity anomaly differences as function of d/o of spherical harmonics: a – before SEM; b – after SEM

the data of ten (10) years of observations by GRACE and those of GOCE complete mission (Bettadpur et al., 2015). Beyond d/o 180, the RMS values for EGM2008 decrease gradually, those of GECO become stable, EGM2008 presents the best performance and EIGEN-6C4 is better than GECO. The RMS values of differences for pure satellite models are increasing exponentially. Table 4 shows the RMS values of differences for pure satellite GGMs up to d/o 180, 200, and 215 after SEM. The differences of RMS values between the models range from 0.01 to 0.02 mGal, at this level, DIR5 always has the smallest RMS value for the considered d/o. Abd-Elmotaal (2015) also found out that DIR5 better fits to the gravity field in Africa.

Table 4. RMS of gravity anomaly differences in mGal for pure satellite GGMs up to d/o 180, 200 and 215 after SEM

d/o	DIR5	SPW5	TIM5	EIGEN-6S4v2	GGM05G	ITU_GGC16
180	4.99	5.00	4.99	4.99	5.00	4.99
200	5.04	5.06	5.04	5.04	5.05	5.04
215	5.08	5.09	5.08	5.08	5.09	5.08

2.3. Comparison of pure satellite GGMs with GNSS/Levelling points

2.3.1. Before SEM

RMS values of geoid height differences in Niger as function of d/o prior to SEM are shown in Figure 6a. All models exhibit similar behaviour, except EGM2008 which

deviates by about 2 cm. Minimum values are reached at d/o 155, 180, and 230. From d/o 190 to 200, GECO and EIGEN-6C4 are better. From d/o 200 to 230, the pure satellite models, excepted GGM05G, are better than the combined models. The best performance of the pure satellite models is observed at d/o 230. At d/o 280, TIM5 and ITU_GGC16 are better, DIR5 and EIGEN_6S4 are better than SPW5 beyond this d/o. Up to d/o 300, EIGEN-6C4 is better than GECO and EGM2008.

The results in northern Nigeria are shown in Figure 6b. The models are also similar until d/o 200 at a cm level. At d/o 155, the minimum RMS value is reached for all models except for EGM2008, ITU_GGC16 gives the smallest value, 0.84 m, while the largest value is shown by EGM2008. From d/o 200 to 300, the combined models show the best performance and EGM2008 is better than GECO and EIGEN-6C4. From d/o 200 to 240, DIR5 and ITU_GGC16 are better than other pure satellite models. From d/o 240 to 280, ITU_GGC16 is better among the pure satellite GGMs, also DIR5 and TIM5 are similar. Beyond d/o 280, DIR5 is better than EIGEN_6S4. SPW5 has the largest RMS values of difference between d/o 240 and 300. Overall, the differences of geoid heights between GNSS/Levelling points and GGMs in Niger are less than those obtained in northern Nigeria.

All pure satellite and combined models containing GOCE data showed a better performance compared to EGM2008 up to d/o ranging from 100 to 200. This confirms the improvement in the gravity field recovery in these spectral bands by GOCE gravity mission.
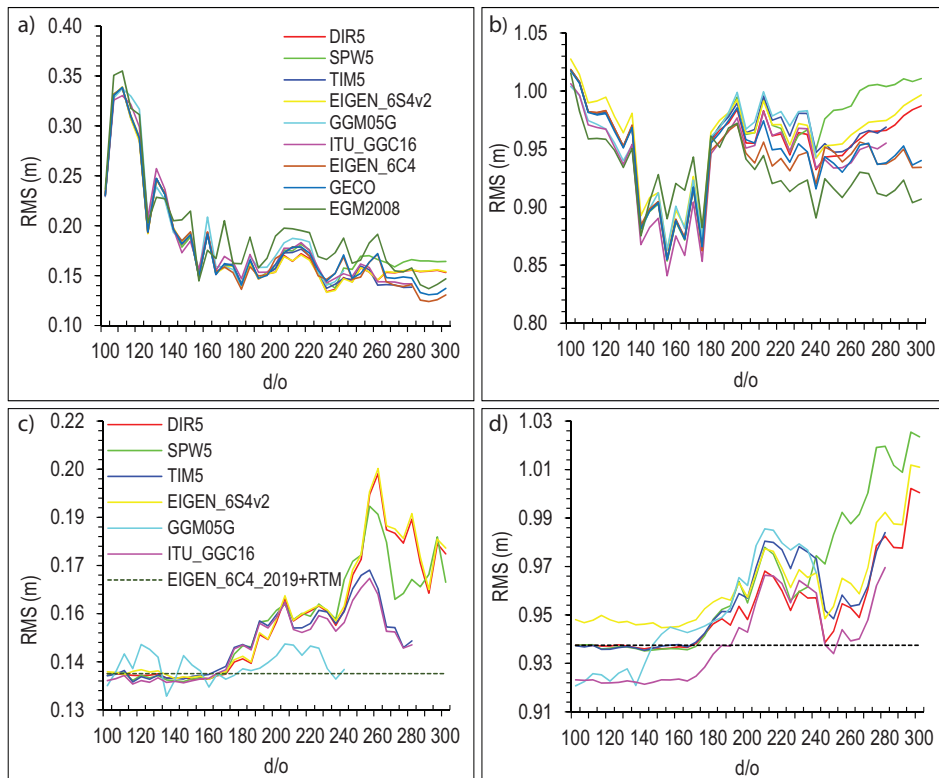

Figure 6. RMS of geoid height as function of d/o of spherical harmonics before SEM:
a – in Niger; b – in Northern Nigeria; and after SEM: c – Niger; d – in Northern Nigeria

2.3.2. After SEM

RMS values of geoid height differences as function of d/o after SEM in Niger are shown in Figure 6c. All models behave similarly at the centimetre level until d/o 200 from which GGM05G presents the smallest RMS value. From d/o 170, RMS values become larger than that of EIGEN-6C4 enhanced by RTM. The results in northern Nigeria are shown in Figure 6d. They are similar to those in Niger, even though the differences are more important in this area. ITU_GGC16 present the smallest RMS values and EIGEN_6S4v2 the worst performance until d/o 200. Table 5 presents RMS values of geoid height differences for pure satellite GGMs at 200. In Niger, the smallest RMS value was observed with GGM05G (0.14 m) and the greatest with SPW5, the other models are similar. ITU_GGC16 showed the smallest RMS value (0.94 m), DIR5 and SPW5 are similar and the greatest value was observed with the remaining GGMs in northern Nigeria. In the two areas, ITU_GGC16 and DIR5 always perform better. As for gravity anomalies, DIR5 is suitable for modelling the long and medium wavelengths of the geoid in Niger.

Table 5. RMS of geoid height differences for pure satellite GGMs in metres at d/o 200 after SEM

Area	DIR5	SPW5	TIM5	EIGEN_6S4v2	GGM 05G	ITU_GGC16
Niger	0.15	0.16	0.15	0.15	0.14	0.15
Nigeria	0.95	0.95	0.96	0.96	0.96	0.94

Conclusions

In this study, we assessed recent GOCE-based GGMs and EGM2008 in Niger Republic and surrounding areas. The quantities derived from combined GGMs EIGEN_6C4, GECO and EGM2008 evaluated up to their maximum d/o 2,190 were compared to free air gravity anomalies from terrestrial data and geoid undulations from GNSS/Levelling points before and after applying SEM by RTM. We found out that the differences of gravity anomalies do no follow normal and the probability according to the 3-sigma rule is near 99% for all models. In terms of RMS values of differences and at 0.1 mGal order, EGM2008 is the best combined GGMs and GECO showed the worst performance before and after SEM. The combined GGMs showed a sensitivity of the gravity anomalies to the very short wavelengths modelled by RTM, the improvement was observed in terms of mean values of the differences and the probability according the 3-sigma rule. EIGEIN_6C4 always gave the greatest probability value and further showed better performance than EGM2008 in long and medium wavelengths because it contains GOCE data. In the comparison of geoid undulations, the RMS of the differences in metres are: EGM2008 (0.16 and 0.92), EIGEN_6C4 (0.14 and 0.94) and GECO (0.14 and 0.95) respectively in Niger and northern Nigeria. EIGEN_6C4 showed better performance in Niger and EGM2008 in

northern Nigeria. In GNSS/Levelling data areas, there is no improvement of geoid heights by RTM due to low lying terrain in general. We selected EIGEN_6C4 instead of EGM2008 for the SEM of pure satellite models and the densification of the gravity database in Niger.

The comparison of pure satellite GGMs to terrestrial gravity data, before SEM by EIGEN_6C4 and RTM, showed similar performance of all models, the values of RMS decreased with higher d/o until 250 where they became stable. Those of combined models continued decreasing. This finding confirmed the performance pure satellite GGMs only in long and medium wavelengths. After applying SEM, all GGMs behaved similarly to EIGEN_6C4 enhanced by RTM from d/o 100 to 180, except EGM2008 and GGM05G which showed the worst performance. At d/o 200, DIR5 showed the smallest value of RMS. Beyond d/o 180, the combined GGMs are better than the pure satellite ones.

The comparison between GGMs and GNSS/Levelling points before SEM showed also that GOCE-based are better than EGM2008 in long and medium wavelengths in Niger, and in northern Nigeria only up to d/o 200. After applying SEM, GGMs showed similar performance at 1 cm lever in Niger until d/o 180, where RMS values became larger than that the enhanced EIGEN_6C4 and GGM05G showed the best performance. In northern Nigeria ITU_GGC16 presented the best performance. As with the gravity anomalies, DIR5 always showed small values of RMS at d/o 200, respectively 0.14 m and 0.95 m in Niger and northern Nigeria. We then selected DIR5 for modelling the long and medium wavelengths of the gravity anomalies and geoid in Niger.

This study confirmed the improvement in the gravity field recovery by GOCE spatial mission in the long and medium wavelengths. It is the first of its kind, carried out in Niger. As the main scientific contribution, we showed the performance of EGM2008 and recent GOCE-based models in this area of the African continent. This is also the result of efforts to collect sufficient data for this type of applications. The study provided an insight on the expected accuracy in GNSS levelling using the GGM-derived geoid models in the area of interest. However, efforts remain to be made in order to collect more GNSS/levelling points.

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