Invited Review

SUPPORT VECTOR MACHINES IN STRUCTURAL ENGINEERING: A REVIEW

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Abstract. Recent development in data processing systems had directed study and research of engineering towards the creation of intelligent systems to evolve models for a wide range of engineering problems. In this respect, several modeling techniques have been created to simulate various civil engineering systems. This study aims to review the studies on support vector machines (SVM) in structural engineering and investigate the usability of this machine learning based approach by providing three case studies focusing on structural engineering problems. Firstly, the concept of SVM is explained and then, the recent studies on the application of SVM in structural engineering are summarized and discussed. Next, we performed three case studies using the experimental studies provided. Applicability of SVM in structural engineering is confirmed by these case studies. The results showed that SVM is superior to various other learning techniques considering the generalization capability of produced model.

Keywords: support vector machines, statistical learning, structural engineering, ultimate load capacity, FRP reinforcement, SFRC corbels, haunched beams.

Introduction

Recent advance in data modeling technology has drawn great attention from various engineering practices. As a result, empirical data modeling is becoming more significant to researchers and engineers in practice. Thus, a process of data training is used to create a model of a system for the purpose of obtaining predictions for the cases that are yet to be observed. The model performance, therefore, is largely dependent on the quantity and the accuracy of experimental findings used for training the model. Being a new modeling technique, support vector machines approach (SVM) is one of those methods that can be implemented to predict test results.

In structural engineering, empirical data modeling is of great concern as it helps researchers and engineers to predict test results. That is, this research attempts to investigate the applicability of this novel approach to structural engineering problems. This paper has five parts. First, it explains the historical development and the key principals of support vector machines (SVM). Then, the applications of SVM on various structural engineering problems are reviewed. Next, three case studies are performed to confirm the performance of SVM. The paper concludes with a discussion of provided case studies' results and applicability of this new approach on structural engineering problems.

1. Support vector machines as a modeling tool

SVMs were first formulated and presented by Boser, Guyon and Vapnik (Boser *et al.* 1992), and were presented for the first time at the Computational Learning Theory (COLT) conference in 1992. Main characteristics of the approach were already available in the literature and were applied in machine learning like large margin hyper planes in the input space since the 1960s (Cristianini, Shawe-Taylor 2000).

After being presented, many researchers have processed both the algorithmic and theoretical analysis of these systems, producing in just a few years what is efficiently a novel research path in its own right, merging subjects of scientific research areas such as statistics, optimization, functional analysis as well as machine learning. A few years later, Cortes and Vapnik (1995) presented the soft margin classifier. The algorithm was expanded to the regression case in 1995 (Vapnik 2000).

Besides its solid numerical basis in statistical learning theory, support vector machines have showed extremely competing performance in several applications,

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e.g., face recognition, text mining, bioinformatics and image processing. This fact has proven that SVMs are one of the state-of-the-art approaches for data mining and machine learning, together with some other soft computing methods, e.g. fuzzy systems and neural networks (Wang 2005).

Being a controlled learning technique, the SVM yields input-output mapping functions from a set of labeled training data. Either a regression or a classification function, i.e. the category of the input data can be the mapping function. Nonlinear kernel functions, for classification, are frequently applied to convert input data to a high-dimensional feature space. By this way, the input data become more separable in comparison to the exact input space. Creation of maximum-margin hyper planes is then performed (Wang 2005).

By solving the optimization, the parametric quantities of the maximum-margin hyper plane are derived. There exist numerous specific algorithmic rules for the purpose of rapidly solving the QP problem that originates from SVMs, generally dependent on heuristic rules to break the problem down into smaller, more controllable portions. Piatt's SMO algorithm is a typical technique for solving the QP problem. This algorithm breaks the problem down into 2D sub-problems for the purpose of analytical analysis. Thus, the need for a numerical optimization algorithm, i.e. conjugate gradient method is eliminated.

The model then developed depending on solely a subset of the training data adjacent to the class boundaries. Likewise, the model created by Support Vector Regression neglects any training data that is adequately close to the model prediction. It is supposed that the SVMs are likewise belonging to "kernel techniques" (Wang 2005).

The development order of SVMs has been reverse to the evolution of NNs. SVMs were developed from the sound theory to the application and tests, while the NNs were accompanied by trial-and-error path, from implementations and considerable experimentation to the theory. Surprisingly, SVMs has not been widely appreciated at the beginning despite its very strong theoretical background. Nowadays, it is obvious that SVMs demonstrate better (or comparable) performance than neural networks and other statistical models, on solving the real world problems (Huang *et al.* 2006).

1.1. Support vector classification

SVMs are quite useful for data classification purposes. The classification process starts with separating data into training and testing sets. Each data row in training set includes a target value and several "attributes". The main purpose of SVM is to create a model which predicts the target values of the test data by using given attributes (Hsu *et al.* 2003). The kernel functions used for data classification are as follows:

– Linear:

$$K(x_i, x_j) = x_i^T x_j;$$

– Polynomial:

$$K(x_i, x_j) = (\gamma x_i^T x_j + r)^d, \ \gamma > 0;$$

– Radial Basis Function (RBF):

$$K(x_i, x_j) = \exp(-\gamma \left\| x_i^T x_j \right\|^2), \ \gamma > 0;$$

– Sigmoid:

$$K(x_i, x_j) = \tanh(\gamma x_i^T x_j + r).$$

where γ, d and r are kernel parameters.

Normally, the procedure of SVM includes:
1. Data transformation to the format of SVM package;
2. Trying γ a few kernels and parameters on a random basis;
3. Testing the model.

Hsu *et al.* (2003) proposed a new SVM procedure as explained below:
1. Data transformation to the format of SVM package;
2. Application of simple scaling on data;
3. Consider RBF kernel function;
4. Obtain the values for parameters C and γ by using cross-validation;
5. Use obtained C and γ parameters to train the whole training set;
6. Testing the model.

1.2. Support vector regression

SVM were initially evolved for solving the classification problems. However, researchers started utilizing SVM to solve regression problems by preserving the entire maximal margin algorithm. A function named as-insensitive loss function that neglects errors that are inside a definite distance of the exact value is able to supervise a parametric quantity that is equal to the margin parameter for separating hyper planes. In SVR (support vector regression), for a given set of training data, the main goal to obtain a function that has maximum difference from the exact found targets for all the training data, and at the same time is at most flat, i.e. we do not focus on errors as long as they are less than, but any deviation larger than a certain amount is not acceptable (Chen *et al.* 2004). The (linear) – insensitive loss function $L(x, y, f)$ is described as:

$$L^\varepsilon(x,y,f) = |y - f(x)|_\varepsilon = \begin{cases} 0 & if \ |y - f(x)| \le \varepsilon \\ |y - f(x)| - \varepsilon & otherwise \end{cases},$$
$$(1)$$

where f is a real-valued function on a x and the quadratic ε-insensitive loss is defined by:

$$L_2^\varepsilon(x,y,f) = |y - f(x)|_\varepsilon^2. \quad (2)$$

Figure 1 illustrates the form of linear and quadratic ε-insensitive loss function for zero and non-zero ε.

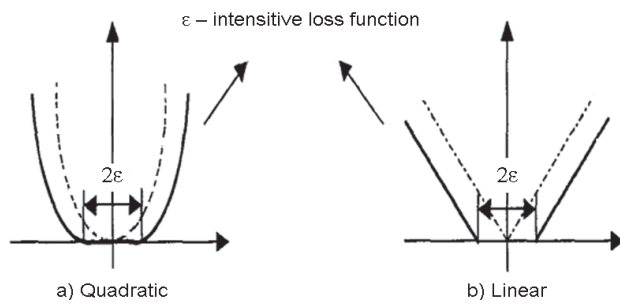The loss function $L(y, f(x,\omega))$ determines the performance of accuracy. Performing linear regression in the

Fig. 1. The form of linear and quadratic ε-insensitive loss function for zero and non-zero ε

high-dimension feature space by the use of ε-insensitive loss function, SVM attempts to decrease model complexity by performing the minimization of $\|\omega\|^2$. By introducing (non-negative) slack variables $\xi_j, \xi_i^*\ i = 1, ..n$:

$$L(y, f(x, \omega)) = |y - f(x)|_\varepsilon^2 ;$$
$$L_2^\varepsilon(x, y, f) = |y - f(x)|_\varepsilon^2 \tag{3}$$

to determine the deviation of training data outside $\varepsilon -$ zone. Following formulation is utilized for the minimization of SVM regression:

$$\frac{1}{2}\|\omega\|^2 + c\sum_{i=1}^{n}(\xi_{i+}\ \xi_i^*) \text{ subject to } \xi_j, \xi_i^*\ i = 1, ..n; \tag{4}$$

$$\xi_j, \xi_i^*\ i = 1, ..n. \tag{5}$$

The solution of this optimization problem can be found by transforming it into the dual problem:

$$f(x) = \sum_{i=1}^{n_{sv}}(\alpha_j - \alpha_i^*)K(x_j, x) \text{ subject to}$$
$$0 \le \alpha_i^* \le C, 0 \le \alpha_j \le C, \tag{6}$$

where n_{sv} is the number of support vectors (SVs) and $K(x_j, x)$ is a kernel function. Generalization capability (accuracy of estimation) of SVM is dependent on a proper setting of meta-parameters C, ε and the kernel parameters. Current software applications usually allow users to define meta-parameters of SVM regression (Cherkassky, Ma 2002).

Parameter C controls the exchange between the model complexity as well as the degree to which deviations larger than ε are tolerated in optimization formulation. Parameter ε describes the width of ε − insensitive zone, which is utilized to fit the training data. The number of SVs used to create the regression function can be affected by the value of ε. The fewer SVs are obtained by choosing the bigger ε. On the other hand, greater ε -insensitive values cause more 'flat' predictions. Although in different ways, both C and ε values affect model complexity (flatness) (Cherkassky, Ma 2002).

There are numerous kernel functions for machine learning. In this study, four different kernel functions namely as linear, polynomial, radial basis and sigmoid functions are used:

– Linear kernel function:

$$K(x_i, x) = x_i x; \tag{6a}$$

– Polynomial kernel function:

$$K(x_i, x) = (x_i(x + 1))^d ; \tag{6b}$$

– Radial basis function:

$$K(x_i, x) = \exp\left[-\frac{(x_i - x)(x_i - x)}{2\sigma^2}\right]; \tag{6c}$$

– Sigmoid kernel function:

$$K(x_i, x) = \tanh(x_i(x + 1)), \tag{6d}$$

where x_i and x are the training and test patterns, respectively, σ is the global basis function width and d is an input vector dimension.

Wu *et al.* (2012) studied the hybrid kernel function, which is a combination two kernel functions namely as linear kernel and local kernel function (RBF). The results of SVM models, which are created using hybrid kernel function, perform better than those created using ordinary functions.

2. Review of SVM applications in structural engineering

2.1. Modeling the material properties of concrete using SVM

Cement concrete is one of the most popular materials used in structural engineering. Modeling and predicting the concrete parameters has been a difficult task since the concrete contains several compounds namely as cement, sand, aggregate, water and admixtures and because of the nonlinearity between the relationships of these dependent and independent compounds. Support vector machines are one of the computational tools that attempt to solve these complex real life problems. The fallowing parts give detailed review of literature concerning the application of SVM in modeling the parameters of various types of concrete.

2.1.1. Applications in modeling the strength of concrete

Concrete strength is a substantial characteristic as it has a strong relationship with the quality and strength of overall structure. Several researchers implemented support vector machines approach in prediction of concrete strength successfully. Shengguo and Junbo (2007) used two types of kernel functions and found that the predictions of developed SVM model match fairly well with those of experiments.

Xu *et al.* (2008) established a SVM-based mathematical model to investigate the relation between the strength and physical quantities that they obtained from nondestructive testing. They discovered that this method has advantages such as less computational time and high precision in predictions. This new method can be used for concrete strength conversion for nondestructive examinations.

High temperature causes serious damage in concrete structure. The nonlinearity of the relationship between material properties of concrete and fire exposure induces the difficulty of correct estimation of exposed temperature. Chen *et al.* (2009) produced a SVM model that is fairly feasible in predicting the exposed temperature values and they also underlined the importance of the numbers of effective parameters.

An example of strength prediction is provided by the study of Hai-xia (2010) who implemented support vector machines approach to predict the strength of high strength concrete. They compared the SVM results to those of actual experiments as well as regression, back-propagation (BP) network and radial basis function (RBF) network. They also emphasized the capability of SVM in prediction of concrete strength.

Prediction of elastic modulus is also a significant property as it is used to calculate deformation of structures. As an example of this, Yan and Shi (2010) proposed a SVM model in order to investigate the applicability of support vector machines in predicting elastic modulus of normal and high strength concrete. They also performed a comparison of SVM results and other proposed models as well as experimental outcomes. They concluded that the performance and generalization capability of SVM model are superior to those of other models.

Bin *et al.* (2011) produced a model to estimate the concrete compressive strength of two sets cubic samples one of which contains fly ash whereas the other contains fly ash with slag ash admixtures. It is stated that the model performed better prediction accuracy than the linear regression method and is more stable than the BP artificial neural network.

Integrated approaches are also effective when predicting concrete properties. An example of this is provided by the work of Gilan *et al.* (2011) who predicted the concrete compressive strength with support vector regression by adopting evolutionary fuzzy function. They compared the robustness and the generalization capability of evolutionary fuzzy function with support vector regression (EFF-SVR) model with some existing modeling approaches such as adaptive neural-fuzzy inference system (ANFIS), fuzzy function with least squared estimation (FF-LSE), artificial neural network and improved FF-LSE. The results indicated the better performance and capability of EFF-SVR model.

Modeling of reinforced and prestressed concrete deep beams is presented by Pal and Deswal (2011) who implemented SVM to predict the shear strength. For comparison, they also applied a back-propagation neural network and three empirical equations for deep beams. The results revealed that the SVM model performs better and the parametric studies exhibit the importance of effective parameters such as concrete strength and ratio of shear span to effective depth of beam on strength prediction.

Wang *et al.* (2011) proposed a model based on SVM in order to forecast the concrete strength and feasibility. The results suggested that SVM is a better approach to express the relationship between concrete strength and effective factors.

Sriraam *et al.* (2012) employed the SVM for investigating the feasibility of using granite fines as a replacement for river-bad sand in concrete. Results indicated that the implementation of SVM induces high performance in predicting the concrete strength.

The use of robust models is required for estimating the compressive strength of no-slump concrete whose properties and constituents are reasonably sensitive. Sobhani *et al.* (2013) developed a SVM model to predict the compressive strength of no-slump concrete and compared results with optimized neural network (ANN) model. It is stated that although both models exhibited high accuracy in prediction and generalization capability, SVM was very rapid comparing to ANN models.

Sun *et al.* (2013) implemented least squares support vector machine (LS-SVM) for predicting the strengths of concrete specimens with high volume fly ash admixture. They stated that LS-SVM model could avoid the drawbacks of neural networks (ANN) namely as over-training and weak generalization capability.

The relationship between the tensile strength and the compressive strength is studied by Yan *et al.* (2013) who produced a SVM model using the experimental data available in literature. The results are compared to those from experimental studies and are found to be in a good agreement. The results are compared to outcomes of empirical design equations and various models. It is stated that the SVM has a strong potential for estimating the splitting tensile strength from compressive strength.

Another example for modeling the concrete strength is the study of Yang and Dong (2013), they employed SVM to investigate the feasibility of this approach. The results are compared to those of generalized regression neural network method.

Yuvaraj *et al.* (2013) developed a fracture mechanics based SVM model to estimate the features of high strength and ultra-high strength concrete beams. The model attempted to predict the values of fracture energy, critical stress intensity factor, critical crack tip opening displacement and failure loads. The results obtained from produced models were in a good agreement with those of the experimental values.

2.1.2. Predicting the concrete corrosion

Durability of structures is of great concern in structural engineering and is affected significantly by the corrosion

of concrete and reinforcing steel. In this regard, predicting the corrosion severity becomes a substantial phenomenon for the researchers.

An example to prediction of corrosion severity is provided by Shujian *et al.* (2007) who implemented support vector machines as a classifier of corrosion severity. It is stated by the researchers that the method has advantage of high accuracy in classification.

Zhang and Song (2012) conducted a test of fly ash concrete specimens in sulfuric acid and modeled the results by means of support vector machines approach. Comparing the results with measured values, it is implied that this new method of computation has capability to predict the sulfuric acid corrosion of concrete.

The mechanical properties of corroded reinforced concrete are investigated by Yang *et al.* (2014) who conducted tests on specimens under repeated loads. Using support vector machines, the deflection and maximum crack width parameters are predicted and compared to those of test results. A high prediction accuracy of the model is observed.

2.1.3. Applications in modeling self-compacting concrete (SCC) properties

Self-compacting concrete (SCC) is a type of concrete that requires no mechanical compaction and is able to take the shape of the formwork without vibrators. This feature makes it popular as it reduces labor costs significantly and accelerates the construction progress. Following studies performs the estimation of SCC material properties.

Siddique (2008) investigated the potential of SVM approach in predicting the compressive strength and slump flow of self-compacting concrete. The results are compared with those of back propagation neural network model and it is observed that the SVM has a greater performance both for compressive strength and slump flow prediction.

A predictive model based on SVM is constructed by Cao *et al.* (2013) in order to predict the elastic modulus of self-compacting concrete. Through the analysis and comparison with test results, it is proved that this modeling technique is useful and powerful.

Another example of the prediction of SCC properties is the work of Aiyer *et al.* (2014) who examined the capability of least squares support vector machines (LS-SVM) for predicting the compressive strength of self-compacting concrete. The results indicated that the LS-SVM model performed better than ANN.

2.1.4. SVM for structural reliability

The uncertainties related to material characteristics and geometry conditions commonly exist in structural engineering. These uncertainties can impact the design performance significantly and result in serious complications. In this regard, structural reliability analyses are performed to detect the effect level.

Li and Lu (2007) proposed two SVM-based approaches for structural reliability analysis. The approximation of implicit performance function is provided by the implementation of SVM. Comparisons among the proposed reliability analysis and classical ones implied that the SVM method has capability both in performance function and failure probabilities.

2.1.5. Applications in prediction of concrete carbonation

Although it is not as much as corrosion, concrete carbonation is also a serious threat for structural durability. Thus, the estimation of carbonation level is of significant importance structural engineering practice.

As an example to this, Zhitao *et al.* (2008) employed SVM to forecast concrete carbonation level. The comparison of results with those of BP network suggested that SVM method could perform much better and could be a useful reference for application in different engineering disciplines.

The carbonation depth of prestressed concrete is investigated by Can (2009) who constructed a SVM model to perform the analysis. Two different kernel functions namely as wavelet kernel function and radial kernel function are used in SVM analysis. Xiang (2009) also studied the prediction of carbonation depth in concrete by means SVM. The results are compared to those from neural network modeling and it is observed that the SVM has higher accuracy and generalization capability.

2.2. Damage identification

Damage assessment has drawn a great attention from several engineering practitioners in recent years (Zhang *et al.* 2010a). SVM is a newly emerging method in damage identification and classification of the structures.

C.-C. Liu and J. Liu (2010) constructed a SVM based model for damage identification of a long-span arch bridge considering the variation ratio of curvature mode. The precision of the used model is verified by comparing the results with those from RBF neural network.

Another example of damage identification is the study of Tesfamariam and Liu (2010) who conducted eight different statistical damage classification methods one of which is support vector machines. It is observed that the performance of SVM model was one of the best among others.

Zhang *et al.* (2010a) stated, in their paper, that the performance of SVM in predicting the grouting quality of grouted beams is quite accurate in comparison to ANN's. They obtained the experimental results after nondestructive tests they conducted for grouting quality.

A SVM-based model for pattern identification of structural damage is presented by Zhang *et al.* (2010b) who performed the analyses by using wavelet kernel function of SVM. It is found that SVM model is able to overcome the problem of other traditional methods that are insensitive to natural frequencies and other dynamic parameters.

Gao *et al.* (2012) utilized least squares support vector machines (LS-SVM) for damage diagnosis of concrete girder in order to determine damage degree and location. Kernel parameters are optimized to improve the generalization capability of the model. High identification precision by LS-SVM model is observed.

Another example of damage identification in concrete samples is provided by Xie *et al.* (2013) who employed the SVM approach to predict the location of voids based on ground penetrating radar (GPR) technique. The study showed that the SVM exhibits promising performance in the GPR identification of voids in concrete.

Zhu and Hao (2007) employed wavelet support vector machines for structural health monitoring. The response signals of a structure under impact load are collected and turned into wavelet packet components. Then, the energies of these components are calculated as feature vectors. SVM is utilized for training and classification based on these feature vectors. It is stated that the method can be used reliably for damage monitoring.

Zhang *et al.* (2006) showed the ability of support vector regression based approach for structural health monitoring of large-scale structures. According to authors, SVM can reduce the computation time and give accurate results. Also, an on-line SVR-based identification approach is presented to identify nonlinear structural parameters (Zhang, Sato 2006).

Satpal *et al.* (2013), in their study, investigated the effectiveness of SVM in health monitoring of beam-like structures. The results indicated the high performance of SVM without considering the health state of data used. Cheng and Hoang (2014c) utilized LS-SVM method for the purpose of risk score inference for bridge maintenance. The findings demonstrated that the method is successful also for decision-making purposes.

2.3. Miscellaneous applications of SVM in structural engineering

Yinfeng *et al.* (2008) proposed a SVM-based two-stage method to estimate and simulate the nonlinear dynamic response of structures. SVM is used during the second stage wherein the predicted linear responses and excitation is employed to approximate nonlinear mapping. The results demonstrated that the method can be used conveniently to predict the nonlinear structural response.

Samui and Kim (2012) developed SVM to identify the fracture parameters, critical stress intensity factor and the critical crack tip opening displacement of concrete. The results of the model are compared with ANN model and other empirical equations. It is concluded that SVM is a robust technique to predict fracture parameters of concrete.

In order to estimate the tangential displacement of a concrete dam, Ranković *et al.* (2014) presented a model based on support vector machines. The model is created using the data collected during fourteen years. The comparison of SVM results with experimental data confirmed the high performance capability of SVM approach.

Kromanis and Kripakaran (2014) investigated the structural performance monitoring of bridges that are subjected to thermal factors. They proposed a model based on SVM to predict the relationship between temperature distribution and structural response. The study confirmed the accuracy of this novel application. Also, SVM is applied to several other civil engineering disciplines successfully (Cheng, Hoang 2014a, b; Chou *et al.* 2013; Erdis 2013).

Additionally, other soft computing techniques such as ANFIS and ANN are also applied successfully to structural engineering problems (Amani, Moeini 2012; Cevik 2011; Cevik *et al.* 2012; Sonebi, Cevik 2009). The results of SVM and ANFIS models are found to be close and highly accurate noting that both modeling techniques have similar theoretical background.

3. Case studies

This paper also reports case studies in which we employed SVM in three different structural engineering problems and discussed the results. Emphasis was placed on verification of SVM performance by applying it to three unrelated problems wherein different structural members are experimented. The first example focuses on the punching shear strength of concrete slabs reinforced with fiber-reinforced polymer rebars. Secondly, ultimate load capacity of steel fiber reinforced concrete (SFRC) corbels is modeled by means of SVM. Final example gives insight into SVM-based modeling of reinforced concrete haunched beams.

3.1. Punching strength of FRP-reinforced slabs

In recent years, the use of fiber-reinforced polymer (FRP) in structures has drawn a great attention of engineers since it is a potential replacement of traditional steel and is free of corrosion problem. FRPs can be manufactured in different types such as rebar, plates and sheets. The stress-strain behavior of FRP rebars and traditional steel rebars are illustrated in Figure 2.

There exist several design models and empirical equations for predicting the punching shear capacity of FRP-reinforced two-way slabs as shown in Table 1 (El-Ghandour *et al.* 2003; Hassan *et al.* 2013; Hussein *et al.* 2004; Lee *et al.* 2009; Matthys, Taerwe 2000; Nguyen-Minh, Rovňák
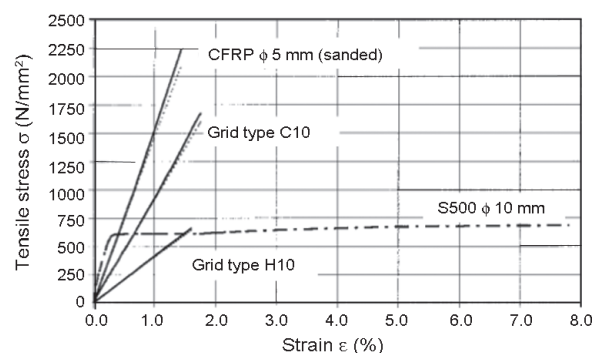
Fig. 2. Stress strain behavior of FRP bars (Matthys, Taerwe 2000)

Table 1. Existing formulas for estimation of punching shear resistance of FRP reinforced two-way slabs

Source	Formula	Remarks
ACI 318-11 (2011) Eqn (8)	$P = 0.33\sqrt{f_c}\, b_{0,0.5d}\, d$	$b_{0,0.5d} = 4(c+d)$
BS 8110-97 (1997) Eqn (9)	$P = 0.79(100\rho)^{1/3}\left[\dfrac{400}{d}\right]^{1/4}\left[\dfrac{f_{ck}}{25}\right]^{1/3} b_{0,1.5d}\, d$	$b_{0,1.5d} = 4(c+3d)$ f_{ck} = cubic compressive strength of concrete
El-Ghandour *et al.* (2003) (ACI) Eqn (10)	$P = 0.33\sqrt{f_c}\left[\dfrac{E_f}{E_s}\right]^{1/3} b_{0,0.5d}\, d$	$b_{0,0.5d} = 4(c+d)$ $E_s = 210\,GPa$
El-Ghandour *et al.* (2003) (BS) Eqn (11)	$P = 0.79\left[100\rho(1.8)\dfrac{E_f}{E_s}\right]^{1/3}\left[\dfrac{400}{d}\right]^{1/4}\left[\dfrac{f_{ck}}{25}\right]^{1/3} b_{0,1.5d}\, d$	$b_{0,1.5d} = 4(c+3d)$ $E_s = 210\,GPa$
Matthys and Taerwe (2000) Eqn (12)	$P = 1.36\dfrac{\left[100\rho\dfrac{E_f}{E_S} f_{cm}\right]^{1/3}}{d^{1/4}} b_{0,1.5d}\, d$	f_{cm} = mean concrete compressive strength of cylinder specimens at 28 days
Ospina *et al.* (2003) Eqn (13)	$P = 2.77\left(\rho f_c\right)^{1/3}\left[\dfrac{E_f}{E_s}\right]^{1/2} b_{0,1.5d}\, d$	$b_{0,1.5d} = 4(c+3d)$ $E_s = 210\,GPa$
Zaghloul (2003) Eqn (14)	$P = 0.07\left(\rho f_c E_f\right)^{1/3} b_{0,0.5d}\, d$	$b_{0,0.5d} = 4(c+d)$
El-Gamal *et al.* (2005) Eqn (15)	$P = 0.33\sqrt{f_c}\,(1.2)^N b_{0,0.5d}\, d\,\alpha$	$b_{0,0.5d} = 4(c+d)$ $\alpha = 0.62(\rho E_f)^{1/3}\left[1+\dfrac{8d}{b_{0,0.5d}}\right]$
Theodorakopoulos and Swamy (2007) Eqn (16)	$P = \dfrac{1}{2}0.234 f_{cu}^{2/3}\left[\dfrac{100}{d}\right]^{1/6}\left[\dfrac{2\alpha_f \lambda_f}{1+\alpha_f \lambda_f}\right] b_{0,1.5d}\, d$ for $\alpha_f > 0.33$ $\alpha_f = \rho_f E_f\left[\dfrac{0.0105}{0.145 f_{cu}}\right]$ with $f_{cu} = f_c/0.8$ $\lambda_f = (k_f/6)(-1+\sqrt{1+48/\alpha_f}) < 1$ with $k_f = 0.55$	$b_{0,1.5d} = 4(c+3d)$ $E_s = 210\,GPa$ f_{cu} = ultimate compressive strength of concrete.

2012; Ospina *et al.* 2003; Zaghloul 2003; Zhang 2006). All of these researchers tested slabs with FRP reinforcing bars or two-dimensional grids. All of the slabs had unrestrained edges and were subjected to a central column load. Carbon (CFRP), glass (GFRP) or a hybrid of carbon and glass (HFRP) types of fibres were used.

In this part of the study, we utilized SVM to estimate the ultimate punching shear load of FRP-reinforced two-way slabs. The main focus is to propose a new model for predicting the punching shear strength of FRP reinforced two-way slabs. The SVM approach is applied to the tests of a concrete member reinforced with composite fibers for the first time in the literature. Thus, an extensive literature review has been conducted on FRP reinforced two-way slabs. Experimental database that is utilized for performing the support vector machine analysis are given in Appendix (Table A.1). The model includes the input variables of column section (c), effective flexural depth of slab (d), elastic modulus of FRP (E_f), reinforcement ratio (ρ), compressive strength of concrete (f_c) and the target variable as ultimate punching load (P).
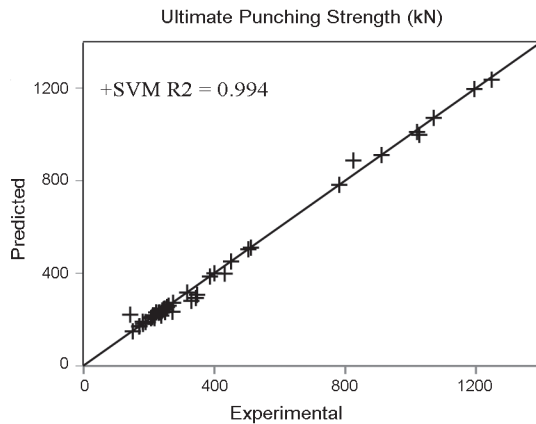
Fig. 3. Performance of proposed SVM model vs. test results

Fig. 4. Main effect trends for the parametric study on punching shear capacity

Fig. 5. Interaction effect plot of variables on punching shear capacity

A process of model creation (8 models) has been performed by using 2 different SVM types (Nu-SVR and Epsilon-SVR) and 4 different SVM kernel functions (Linear, Polynomial, Radial basis function, Sigmoid). 80% of the database is used as training set and the remaining 20% is used for validation in all models. The radial basis function (RBF) model is selected for comparison and parametric study as it exhibited the best performance among others.

According to obtained statistical values, SVM model has the lowest mean squared error (MSE) value of 485.2 whereas the closest MSE is that of Theodorakopoulos and Swamy (2007) with 2260.9. Additionally, SVM model exhibits the highest correlation coefficient (R^2) of 0.994. Test results versus predicted results of SVM model is illustrated in Figure 3. Furthermore, an evaluation process has been performed to test the accuracy of proposed model by means of numerical results of the same experimental database and empirical equations available in the literature. The accuracy of proposed model is found to be higher than numerical results and existing analytical equations and design codes available in the literature.

A parametric study is performed to test the generalization capability of SVM model. For this, a new dataset has been generated for each variable and the ranges of those variables were kept more or less in between the minimum and maximum of variables in experimental dataset. By implementing the generated data to SVM model, main effect (Fig. 4) and interaction plots (Fig. 5) of each variable are obtained. The main effect plot is a significant tool to display the effect of each variable on punching shear capacity. This graphical tool allows viewing the overall importance of variable effects on the output and provides a general snapshot. On the other hand, the interaction plot is an essential tool to observe the effect of each input parameter in conjunction with other inputs. This tool allows viewing the effect of each variable in detail. The same parametric dataset, which is used for main effect plot, has been utilized in order to obtain interaction plot provided in Figure 5.

Main effect (Fig. 4) and interaction plot (Fig. 5) indicate that punching shear capacity of FRP-reinforced two-way slabs is influenced significantly by all input parameters. In particular, c (column section), d (effective flexural depth of slab), E_f (elastic modulus of FRP), f_c (compressive strength of concrete) and ρ (reinforcement ratio) have an increasing effect on punching shear load.
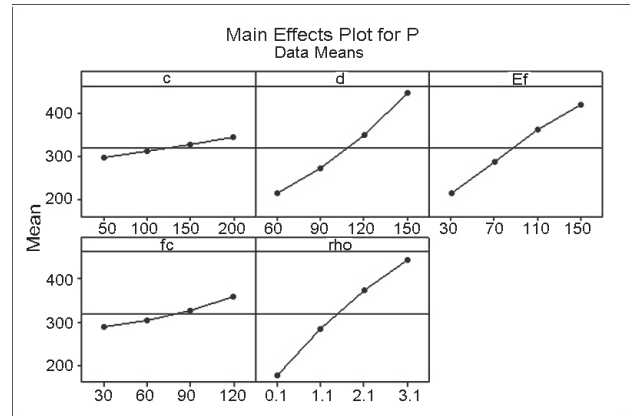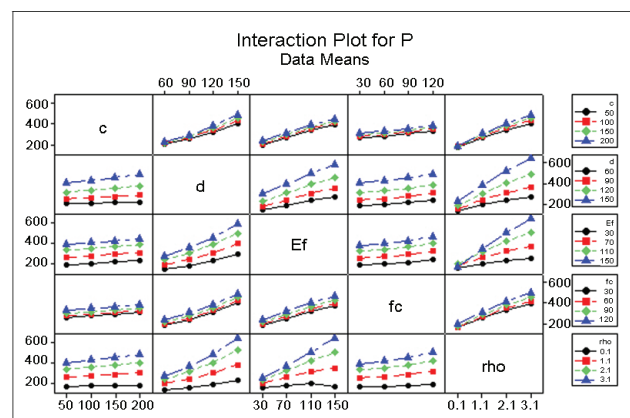
3.2. Ultimate load capacity of Steel Fiber Reinforced Concrete (SFRC) corbels

In the second case study, emphasis was placed on the prediction of ultimate load capacity of SFRC corbels. Corbels are structural elements primarily used in reinforced concrete and precast structures. The main function of corbels is to transfer vertical and horizontal loads to the members to which they are connected. Corbels can be the overhanging portion of beam with a small span length. Shear span to effective depth ratio (a/d) of corbels is less than unity (Ersoy *et al.* 2010).

Use of steel fiber in reinforced concrete corbels provides considerable advantages. Previous studies related with corbels have concluded that reinforced concrete corbels which include only steel fibers as secondary reinforcement have almost the same load carrying capacity with those of horizontal stirrups are used against shear failure.

Use of steel fiber facilitates the fabrication of corbels because of easier placement of it as compared to placement of horizontal stirrups. Steel fibers allow corbels to experience large deflections after achieving ultimate load without a dramatic loss in load carrying capacity or demonstrating a sudden and brittle failure (Fattuhi, Hughes 1989b).

The aim of this study is to present SVM for the prediction and analyses of ultimate load capacity of SFRC corbels for the first time in literature. The outcomes of the study will enable to understand and model the effects of various material and geometry parameters on the ultimate load capacity of SFRC corbels, which has not been studied so far.

Previous experimental studies on SFRC corbels

Fattuhi and Hughes studied a series of experiments on steel fiber reinforced corbels (Fattuhi 1987, 1990a, b, 1994b; Fattuhi, Hughes 1989a, b; Hughes, Fattuhi 1989). Fattuhi and Hughes investigated effects of steel fiber on load carrying capacity of corbels whose test configuration is shown in Figure 6. They changed various parameters (tensile and compressive strength of concrete, volume fraction of steel fiber, shear span, fiber aspect ratio, effective depth, reinforcement ratio) and observed the mechanical response of SFRC corbels.

Campione, Mendola and Mangiavillano studied the flexural behavior of fibrous reinforced corbels experimentally and suggested simple analytical expressions for bearing capacity by considering the shear contribution due to steel reinforcements and fibers (Campione *et al.* 2007).

Fattuhi (1994a) also investigated the mechanical behavior of trapezoidal SFRC corbels. High strength steel fiber reinforced corbels in trapezoidal form were experimented by Muhammad (1998) under monotonic and cyclic loading.
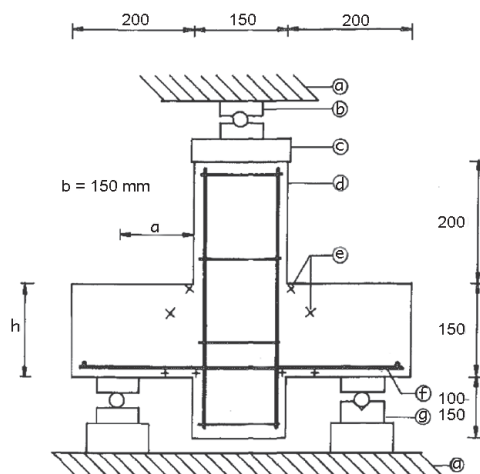
Test Arrangement: (a) Testing Machine; (b) Spherical Seat; (c) Loading Plate; (d) Concrete Specimen; (e) Demec Discs; (f) Main Bars; (g) Roller Support

Fig. 6. Test configuration of corbels experimented by Fattuhi and Hughes (Fattuhi 1990b)

In order to obtain an empirical equation for the prediction of the ultimate load capacity of SFRC corbels tested, Fattuhi proposed the following equation (Fattuhi 1990b):

$$V_n = 57.292bh(f_{ct})^{0.315}\left[\frac{a}{h}\right]^{-0.812}\left[\frac{f_y}{fcu}\right]^{-0.049}\left[\frac{d}{h}\right]^{0.678}\rho^C,$$

(7)

where V_n is the nominal strength of a corbel, f_{ct} is the tensile strength of concrete, a is the shear span (mm), h is the overall depth (mm), f_y is yield strength of main reinforcement and fcu is the cubic compressive strength of SFRC, d is the effective depth (mm) and ρ is the reinforcement ratio (%).

In this part of the study, the main focus is to compare the experimental results of SFRC corbels with the results of newly proposed SVM model. The SVM approach is applied to the experimental results of SFRC corbels for the first time in the literature. An experimental database on SFRC corbels has been gathered from the literature that is utilized for performing the support vector machine modeling (Table A.2). The proposed SVM model includes the input variables of cylindrical compressive strength of concrete (f_c), shear span (a), tensile strength of concrete (f_t), overall depth (h), steel-concrete strength ratio (f_y/f_{cu}), effective depth (d) and reinforcement ratio (ρ). Yet, only most effective parameters are taken when performing the parametric study.

2 different SVR types (Nu-SVR and Epsilon-SVR) and 4 different SVR kernel functions (Linear, Polynomial, Radial basis function and Sigmoid) are utilized to create SVM models. 80% of the database is used as training set and the remaining 20% is used for validation in all models. The model whose performance is the best among others is selected for comparison and parametric study.

Additionally, an evaluation process has been performed to test the accuracy of proposed model by means of numerical results of the same experimental database and an empirical equation (Eqn (7)) proposed by Fattuhi (1990b). The comparison of SVM and empirical equation results compared experimental results are shown in Table A.2. According to obtained statistical values, SVM model has the lowest root mean squared error (RMSE) value of 4.735. Additionally, SVM model exhibits the highest goodness of fit (R^2) by 0.975. Test results versus predicted results of SVM model are illustrated in Figure 7. The accuracy of proposed model is found to be very high and in good agreement with test results.

Main effect (Fig. 8) and interaction plot (Fig. 9) indicate that ultimate shear load capacity of SFRC corbels is influenced by four input parameters namely as compressive strength of concrete (f_c), reinforcement ratio (ρ), fiber density (v_f) and shear span-to-effective depth ratio (a/d). The other parameters are not involved in modeling because of their low contribution. In particular, compressive strength of concrete (f_c), reinforcement ratio (ρ) and
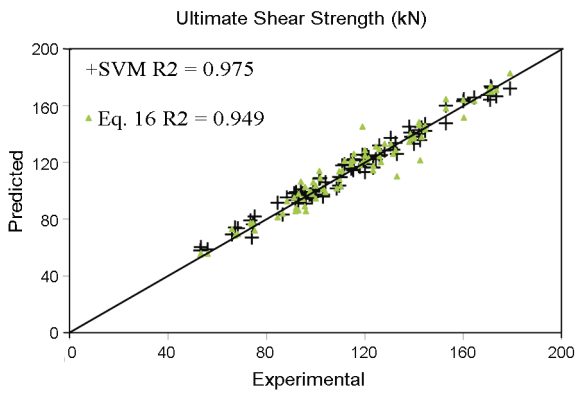
Fig. 7. Experimented versus predicted shear strengths (in KN)
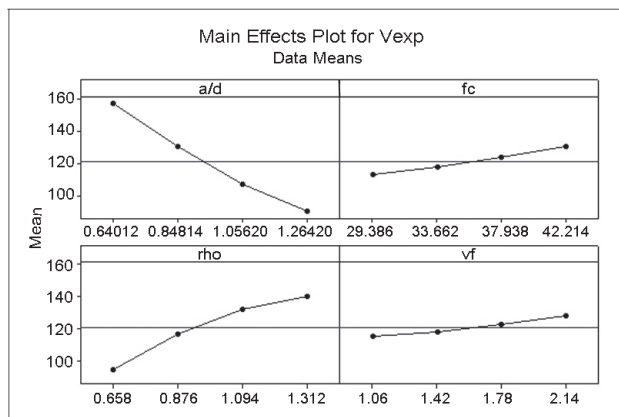
Fig. 8. Main effect trends for the parametric study on ultimate shear force of SFRC corbels
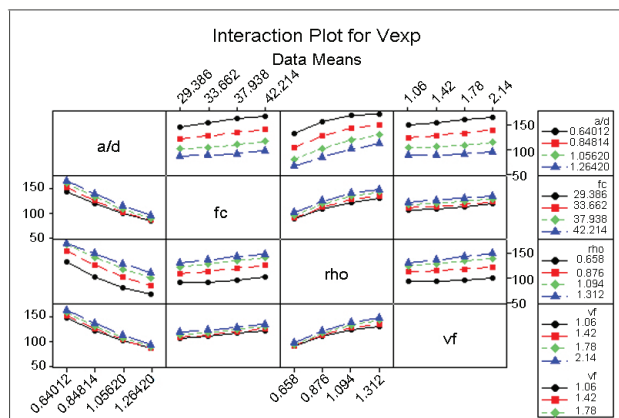
Fig. 9. Interaction effect plot of variables on ultimate shear capacity

fiber density (v_f) have increasing effects on ultimate load whereas the increase in shear span-to-effective depth ratio (a/d) leads to lower load capacity.

It is found that the proposed SVM has generalization capability for the range of variables in experimental dataset. It can be observed from the interaction plots (Fig. 9) that the shear span-to-effective depth ratio (a/d) has a significant inversely proportional effect on ultimate

load and this effect is pronounced more together with the influence of compressive strength (f_c). Although the contribution of fiber density (v_f)) is rather low in comparison with other inputs, the increase rate in fiber density is pronounced more with the contribution of shear span-to-effective depth ratio (a/d) and reinforcement ratio (ρ).

It can be extracted from the parametric study of SFRC corbels that the results of SVM analysis are reliable and can be used in design and analysis process. Because the outputs of main effects plot and interaction plot are consistent with the outputs of experimental and analysis. For instance, the most important parameters influencing the ultimate load capacity of SFRC corbels are span-to-effective- depth ratio (a/d) and reinforcement ratio (ρ) and effect of steel fiber volume ratio is small when compared to these two parameters according to experiments carried by Fattuhi and Hughes. The similar situation exists according to the main effect and interaction effect graphs of SVM analysis.

3.3. Shear capacity of reinforced concrete haunched beams

The third case study focuses on the shear capacity prediction of RC haunched beams. Although reinforced concrete (RC) haunched beams are widely used as bridges or portal frames and precast roof girders, there is a lack of studies in the literature investigating this topic (Nilson *et al.* 2011). Scarce in experimental studies is the main impediment to include this topic in details by international building practice codes. As a result of various experimental studies, it can be concluded that the behavior and failure of the RC haunched beams differs as compared to prismatic section RC beams. Figure 10 illustrates the haunched beams experimented by Debaiky and Elniema (1982), Stefanou (1983), Tena-Colunga *et al.* (2008) and Nghiep (2011).

In this part of the study, SVM approach is implemented to estimate the shear capacity of RC haunched beams. For this, experimental data is collected and used for modeling by means of SVM. The material parameters that are taken to investigate the behavior of the RC haunched beams are shear span-to-critical effective depth ratio (a/d_{cr}), haunch slope ($\tan a$), flexural reinforcement ratio (ρ_s) and compressive strength of concrete (f_c). Characteristics of tested beams and comparison of results are summarized in Table A.3. Figure 11 illustrates the test results versus predicted results of SVM model produced to predict the ultimate shear load of RC haunched beams.

Main effect (Fig. 12) and interaction plot (Fig. 13) show that ultimate shear capacity of reinforced concrete haunched beams is highly influenced by four particular input parameters namely as shear span-to-critical effective depth ratio (a/d_{cr}), haunch slope (*tana*), reinforcement ratio (ρ_s) and compressive strength of concrete (f_c). The other parameters are ignored in modeling because of their low contribution. In particular, reinforcement ratio (ρ), compressive strength of concrete (f_c) and haunch slope
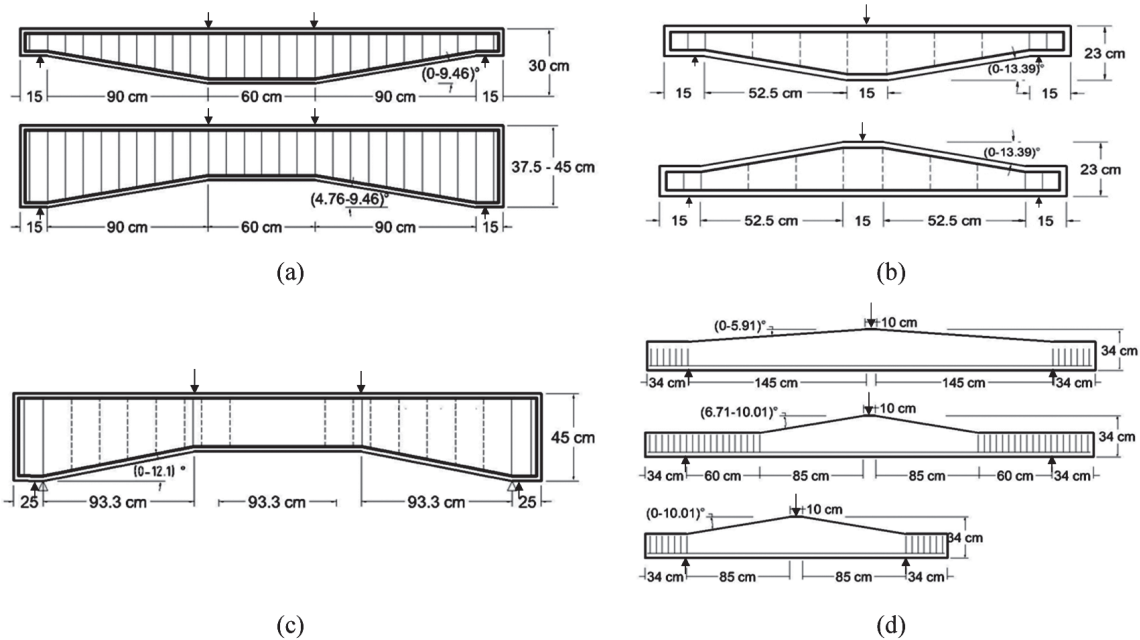
Fig. 10. Haunched beams tested by: (a) Debaiky and Elniema (1982); (b) Stefanou (1983); (c) Tena-Colunga *et al.* (2008); (d) Nghiep (2011)
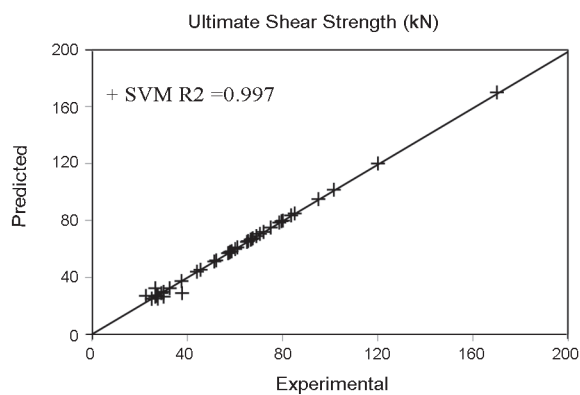
Fig. 11. Experimented versus predicted shear strengths (in KN)
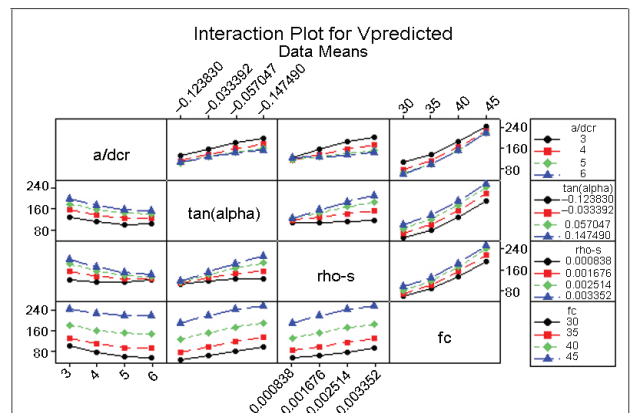
Fig. 13. Interaction plots for the parametric study on ultimate shear force of RC haunched beams
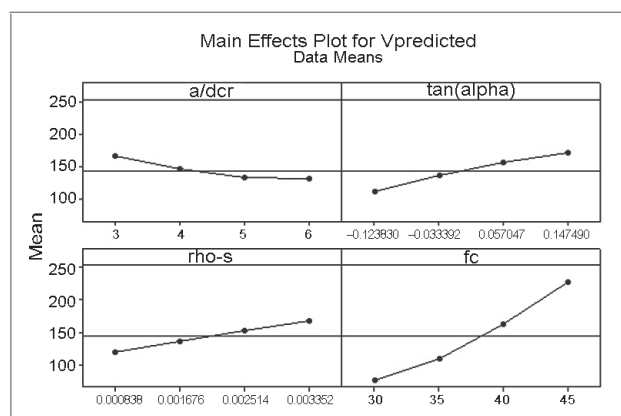
Fig. 12. Main effect trends for the parametric study on ultimate shear force of RC haunched beams

(*tana*) have increasing effects on ultimate load whereas the increase in shear span-to-critical effective depth ratio (a/d_{cr}) leads to lower shear load capacity.

Figures 12 and 13 indicate that the proposed SVM has generalization capability for the range of variables in experimental dataset of haunched beams. The shear span-to-critical effective depth ratio (a/d_{cr}) has an inversely proportional effect on ultimate load and this effect is pronounced more together with the influence of compressive strength (f_c). The effect of compressive strength (f_c) on ultimate load capacity is larger than remaining three parameters. Although the contributions of reinforcement ratio (ρ_s) and haunch slope (*tana*) are rather low, their effects become significant with the contribution of concrete compressive strength (f_c).

4. Advantages of support vector machines

Advantages of support vector machines are as follows:
- The problems of outliers and noise can be minimized by proper setting of C-parameter (Vapnik 2000).
- Regardless of data distribution (whether it is non-monoton or non-linearly seperable), SVMs can exhibit high performance for classification problems and give accurate results (Cristianini, Shawe-Taylor 2000).
- Noisy conditions can be dealt with by automatic identification and incorporation of support vectors as the model minimizes the effect of non-support vectors while training (Han *et al.* 2007).
- Thanks to some key training vectors in the models, future predictions can be improved by tracing back in the historical modeling events (Han *et al.* 2007; Mukherjee, Vapnik 1999).

5. Disadvantages of support vector machines

Disadvantages of support vector machines are as follows:
- The main problem of SVMs is the process of selecting the suitable kernel function and hyper parameters as it based on trial and error process, which is time-consuming (Ccoicca 2013; Yu *et al.* 2006).
- Interpretation and understanding of nonlinear SVR model behavior can be quite difficult because of the fact that the mapping of nonlinear inputs into high dimensional feature space involves high complexity. Thereby, the training process takes more time comparing to those of linear models (Raghavendra, Deka 2014; Tripathi *et al.* 2006).
- Probabilistic forecasting may not be performed since SVMs aim for point predictions (Raghavendra, Deka 2014).
- Since the model is dependent on past data records, model extrapolation performance may be unsatisfactory incase the past data are inconsistent (Raghavendra, Deka 2014).

Conclusions

In this study, the key principles of support vector machines (SVM) and its applications in structural engineering have been reviewed. Additionally, its applicability has been proved by providing three case studies namely as punching shear capacity of FRP-reinforced two-way slabs, ultimate shear capacity of SFRC corbels and the load capacity of haunched beams.

The first case study has been conducted using 47 experimental records of the punching shear capacity of FRP-reinforced two-way slabs. The SVM is found to be robust, highly accurate and capable to generalize for predicting the punching shear capacity. ($R^2 = 0.994$) The second example is performed using 84 experimental data on shear capacity of SFRC corbels. This example also confirmed the robustness and accuracy as well as generalization capability of SVM approach with a correlation coefficient of $R^2 = 0.975$. In this example, the results are also compared

to those of an empirical equation provided in the literature. The third example focuses on the shear capacity of reinforced concrete haunched beams. SVM is applied to predict the ultimate shear capacity using 47 experimental records. Also, the prediction performance of SVM is found to be high ($R^2 = 0.997$) and the generalization capability is confirmed.

Based on the above-mentioned results, fallowing conclusions can be drawn:
- Support vector machines approach has successfully been implemented by several researchers focusing on structural engineering problems.
- SVM approach is a highly effective machine learning method that can be applied to structural engineering problems.
- SVM technique is superior to several other methods considering the high generalization capability performance.
- SVM can safely be implemented to wide range of structural problems even though the problems are highly unrelated.
- As the performance of SVM models are largely dependent on the type of kernel function, it is required to select the suitable kernel function in order to get satisfactory results.
- Encouraging performance of SVM method can draw attention of researchers and engineers who desire to find out the impact rate of inputs on an output of a structural engineering problem.

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Appendices

Table A.1. Experimental database of FRP-reinforced two-way slabs and comparison of models

Source	Specimen	Column shape*	c (mm)	d (mm)	E_f (GPa)	f_c (MPa)	ρ (%)	P_{test} (kN)	P_{test}/P_{pred} Eqn (8)	Eqn (9)	Eqn (10)	Eqn (11)	Eqn (12)	Eqn (13)	Eqn (14)	Eqn (15)	Eqn (16)	SVM
El-Ghandour et al. (2003)	SGI	S	200	142	45	33.3	0.22	170	0.46	0.65	0.76	0.82	1.03	0.87	0.84	0.86	1.00	1.00
	SCI	S	200	142	110	34.7	0.18	229	0.61	0.93	0.74	0.86	1.08	0.79	0.89	0.90	1.06	1.00
	SG2	S	200	142	45	46.6	0.47	271	0.62	0.72	1.02	0.91	1.14	0.96	0.93	0.90	0.99	1.16
	SG3	S	200	142	45	30.3	0.47	237	0.67	0.73	1.10	0.92	1.15	0.97	0.94	0.98	1.00	1.09
	SC2	S	200	142	110	29.6	0.43	317	0.91	1.01	1.11	0.94	1.18	0.86	0.97	1.02	1.05	1.00
Matthys and Taerwe (2000)	C1	C	150	96	91.8	36.7	0.26	181	1.22	1.22	1.58	1.21	1.51	1.25	1.69	1.57	1.42	1.00
	C1'	C	230	96	91.8	37.3	0.26	189	0.95	1.07	1.24	1.06	1.33	1.10	1.33	1.40	1.24	1.00
	C2	C	150	95	95	35.7	1.05	255	1.77	1.11	2.27	1.08	1.36	1.12	1.52	1.43	1.28	1.00
	C2'	C	230	95	95	36.3	1.05	273	1.42	0.99	1.82	0.97	1.22	1.01	1.22	1.30	1.16	1.00
	C3	C	150	126	92	33.8	0.52	347	1.66	1.29	2.15	1.28	1.60	1.24	1.80	1.56	1.42	1.13
	C3'	C	230	126	92	34.3	0.52	343	1.26	1.10	1.63	1.09	1.37	1.06	1.37	1.35	1.22	1.17
	CS	C	150	95	147.6	32.6	0.19	142	1.03	1.12	1.14	0.95	1.19	0.91	1.33	1.26	1.07	0.64
	CS'	C	230	95	147.6	33.2	0.19	150	0.81	1.00	0.90	0.84	1.05	0.81	1.05	1.14	0.95	1.00
	H1	C	150	95	37.3	118	0.64	207	0.79	0.71	1.38	0.95	1.19	1.15	1.33	1.02	1.14	1.00
	H2	C	150	89	40.7	35.8	3.78	231	1.75	0.72	2.98	0.93	1.16	1.13	1.30	1.25	1.17	1.00
	H2'	C	80	89	40.7	35.9	3.78	171	1.83	0.64	3.11	0.82	1.03	1.00	1.36	1.09	1.04	1.00
	H3	C	150	122	44.8	32.1	1.21	237	1.22	0.71	2.00	0.89	1.12	0.98	1.25	1.11	1.00	1.09
	H3'	C	80	122	44.8	32.1	1.21	217	1.50	0.75	2.47	0.94	1.18	1.04	1.55	1.16	1.06	1.00
Ospina et al. (2003)	GFR-1	S	250	120	34	29.5	0.73	217	0.68	0.68	1.23	0.93	1.17	1.08	0.90	1.05	1.03	1.05
	GFR-2	S	250	120	34	28.9	1.46	260	0.83	0.65	1.49	0.89	1.12	1.04	0.86	1.01	1.01	1.00
	NEF-1	S	250	120	28.4	37.5	0.866	206	0.57	0.56	1.10	0.82	1.03	0.98	0.79	0.88	0.91	1.00
Hussein et al. (2004)	GS1	S	250	100	42	40	1.183	249	0.85	0.76	1.43	0.98	1.22	1.14	0.94	1.09	1.10	1.07
	GS2	S	250	100	42	35	1.052	218	0.80	0.72	1.34	0.93	1.17	1.09	0.89	1.06	1.05	1.00
	GS3	S	250	100	42	29	1.671	240	0.96	0.73	1.62	0.93	1.17	1.09	0.90	1.10	1.10	1.00
	GS4	S	250	100	42	26	0.947	210	0.89	0.80	1.50	1.02	1.28	1.20	0.98	1.23	1.17	0.99
Lee et al. (2009)	GFU1	S	225	110	48.2	36.3	1.17	222	0.76	0.65	1.22	0.80	1.00	0.89	0.79	0.88	0.90	0.96
	GFB2	S	225	110	48.2	36.3	2.14	246	0.84	0.59	1.35	0.72	0.90	0.81	0.71	0.80	0.85	1.00
	GFB3	S	225	110	48.2	36.3	3	248	0.85	0.53	1.36	0.65	0.81	0.73	0.64	0.72	0.79	1.00
Zhang (2006)	GS2	C	250	100	42	35	1.05	218	1.02	0.72	1.71	0.93	1.17	1.09	1.14	1.23	1.05	1.00

Continued Table A.1

Source	Specimen	Column shape*	c (mm)	d (mm)	E_f (GPa)	f_c (MPa)	ρ (%)	P_{test} (kN)	P_{test}/P_{pred}								
									Eqn (8)	Eqn (9)	Eqn (10)	Eqn (11)	Eqn (12)	Eqn (13)	Eqn (14)	Eqn (15)	Eqn (16)	SVM
Nguyen-Minh and Rovňák (2012)	GSL-0.4	S	200	129	48	39	0.48	180	0.61	0.61	0.98	0.75	0.94	0.80	0.76	0.79	0.82	0.94
	GSL-0.6	S	200	129	48	39	0.68	212	0.70	0.63	1.12	0.78	0.97	0.84	0.80	0.82	0.86	0.97
	GSL-0.8	S	200	129	48	39	0.92	244	0.63	0.68	1.02	0.83	1.05	0.86	0.77	0.87	0.91	1.00
Hassan et al. (2013)	G(0.7)30/20	S	300	150	48.2	34	0.71	329	0.69	0.76	1.11	0.94	1.17	0.97	0.86	0.95	1.01	1.17
	G(0.7)30/20-B	S	300	150	48.2	39	0.71	386	0.77	0.65	1.25	0.80	1.01	0.83	0.74	0.82	0.90	1.00
	G(1.6)30/20	S	300	150	48.1	39	1.56	431	0.89	0.73	1.44	0.90	1.13	0.93	0.83	0.94	1.02	1.08
	G(1.6)30/20-B	S	300	150	48.1	32	1.56	451	0.50	0.63	0.81	0.77	0.97	0.80	0.64	0.76	0.83	1.00
	G(0.7)45/20	S	450	150	48.2	45	0.71	400	0.75	0.68	1.21	0.84	1.05	0.87	0.69	0.88	0.95	1.00
	G(1.6)45/20	S	450	150	48.1	32	1.56	504	0.69	0.65	1.11	0.79	1.00	0.82	0.66	0.81	0.89	1.00
	G(1.6)45/20-B	S	450	150	48.1	39	1.56	511	0.60	0.81	0.96	0.99	1.25	0.86	0.92	0.87	1.02	1.00
	G(0.3)30/35	S	300	300	48.2	34	0.34	825	0.53	0.73	0.85	0.90	1.13	0.78	0.84	0.77	0.92	0.93
	G(0.3)30/35-B	S	300	300	48.2	39	0.34	782	0.72	0.78	1.16	0.96	1.20	0.83	0.89	0.82	0.98	1.00
	G(0.7)30/35	S	300	300	48.1	39	0.73	1071	0.79	0.82	1.27	1.00	1.25	0.87	0.93	0.89	1.03	1.00
	G(0.7)30/35-B-1b	S	300	300	48.1	30	0.73	1027	0.73	0.82	1.18	1.00	1.26	0.87	0.93	0.83	1.02	1.03
	G(0.7)30/35-B-2b	S	300	300	48.1	47	0.73	1195	0.44	0.70	0.70	0.86	1.08	0.75	0.72	0.71	0.89	1.00
	G(0.3)45/35	S	450	300	48.2	49	0.34	911	0.61	0.91	0.98	1.11	1.40	0.97	0.93	0.98	1.14	1.00
	G(0.3)45/35-B	S	450	300	48.2	32	0.34	1020	0.77	0.88	1.23	1.08	1.35	0.94	0.90	0.96	1.11	1.01
	G(0.7)45/35	S	450	300	48.1	30	0.73	1248	0.88	0.79	1.36	0.92	1.15	0.95	0.99	1.01	1.03	1.01
Mean									0.88	0.79	1.36	0.92	1.15	0.95	0.99	1.01	1.03	1.01**
Std. Dev.									0.35	0.18	0.52	0.12	0.16	0.14	0.28	0.22	0.14	0.08**
CoV									0.4	0.23	0.39	0.14	0.14	0.14	0.28	0.22	0.14	0.08**
MSE									80698.4	24298.5	12790.6	3753.2	9209.3	7314.7	11619.8	10158.7	2260.9	485.2**
R^2									0.893	0.953	0.894	0.968	0.968	0.978	0.945	0.964	0.974	0.994**

Table A.2. Material and geometric properties of analyzed SFRC corbels and comparison of models

References	Corbel Number	a (mm)	b (mm)	d (mm)	h (mm)	ρ (%)	f_c (MPa)	f_t (MPa)	l/d_f	v_f (%)	V_{exp} (kN)	$V_{(Eq.\ 7)}/V_{exp}$	V_{SVM}/V_{exp}
Hughes and Fattuhi (1989)	C2	125	152	120	147.5	0.700	43.34	4.37	48	0.70	84.5	1.039	0.98
	C3	125	152	119	146	0.710	42.61	5.45	80	0.70	92.9	0.993	0.99
	C4	125	151	123	149.5	0.700	41.63	4.79	92	0.70	91.8	1.060	1.01
	C5	125	152	119	146	0.710	41.39	5.36	100	0.70	96.0	1.012	1.08
	C6	125	156	117	146.5	0.690	32.48	3.19	100	0.70	75.2	1.200	0.96
Fattuhi and Hughes (1989b)	C27	52.5	153	121	148.5	0.450	38.31	4.64	92	0.70	125.8	1.034	1.01
	C28	89	151	124	148	0.450	45.12	6.09	92	0.70	88.2	1.008	0.99
	C29	125	153	130	149	0.440	45.12	6.09	92	0.70	65.9	1.032	0.95
	C30	52.5	154	121.5	146.5	0.700	41.63	4.79	92	0.70	171.0	1.115	0.96
	C31	64.5	153	118	146	1.020	46.17	5.05	92	0.70	179.0	1.089	1.06
	C32	125	153	118	148	1.000	38.31	4.64	92	0.70	110.1	1.020	1.01
Fattuhi and Hughes (1989a)	T3	89	152	122	148	0.700	38.80	4.66	92	0.70	133.0	0.996	1.05
	T4	89	151	123	147	0.710	45.28	6.19	92	1.4	142.5	1.018	0.99
	T5	89	152	123	147	0.700	46.49	9.28	92	2.1	143.0	1.059	1.00
	T10	89	151	117	147	1.020	38.80	4.66	92	0.70	138.0	1.086	1.02
	T11	89	152	121	146	1.020	45.28	6.19	92	1.4	160.2	1.073	1.00
	T12	89	152	121	147	1.020	46.49	9.28	92	2.1	171.2	1.033	0.99
Fattuhi (1990a)	1	80	152.5	123	149	1	33.53	5.84	60	1.66	153.0	1.042	1.05
	2	80	155	124	150	0.98	35.15	5.44	60	1.66	160.0	1.025	1.07
	3	80	152.5	126	150	0.44	34.02	4.86	60	1.66	91.2	1.078	0.94
	4	80	155	125	149	0.44	32.89	5.30	60	1.66	93.0	1.064	0.98
	5	140	155	123	149	0.98	32.81	5.46	60	1.66	103.0	0.980	0.99
	6	140	154.5	124	150	0.98	30.78	5.35	60	1.66	95.7	1.047	0.99
	7	140	153	126	150	0.44	27.38	3.89	60	0.74	53.3	1.016	1.04
	8	140	153	125.5	149.5	0.44	29.89	3.72	60	0.74	53.1	1.013	0.99
	9	80	152.5	123	149	1	27.95	5.29	60	1.66	152.9	1.003	0.96
	10	140	155.5	123	149	0.98	30.05	5.24	60	1.66	102.9	0.963	1.00
	11	140	153	126	150	0.44	29.00	3.76	60	0.74	56.0	0.961	1.13
	12	80	154	125	149	0.44	30.78	3.89	60	0.74	92.0	0.972	1.02
	13	110	154.7	123	149	0.99	27.54	5.04	60	1.66	111.7	1.086	1.03
	14	110	153.5	125	149	0.44	29.57	4.24	60	0.74	68.3	0.987	1.06
	15	110	152.5	126	150	0.44	31.59	3.92	60	0.74	67.2	1.045	1.04
	16	110	154.5	123.5	149.5	0.98	30.54	4.94	60	1.66	114.3	1.115	1.02
	18	89	154	124.5	150.5	0.99	26.41	4.98	100	1.00	119.0	1.191	1.08

Continued Table A.2

References	Corbel Number	a (mm)	b (mm)	d (mm)	h (mm)	ρ (%)	f_c (MPa)	f_t (MPa)	l/d_f	v_f (%)	V_{exp} (kN)	$V_{(Eq.7)}/V_{exp}$	V_{SFM}/V_{exp}
Fattuhi (1990b)	20	110	153	123.5	149.5	0.99	31.27	5.43	60	1.75	126.0	1.016	1.08
	21	110	156	122	148	0.98	29.97	4.73	60	1.50	118.0	1.047	1.09
	22	100	153	123	149	0.69	29.97	4.73	60	1.5	108.5	0.987	1.09
	23	110	153	122.5	148.5	1	27.38	5.12	60	2.0	126.5	0.994	1.01
	24	80	153	124	150	0.69	27.38	5.12	60	2.0	131.5	1.022	1.00
	27	80	153.5	123.5	149.5	0.99	34.26	6.29	60	2.5	171.5	1.038	1.01
	28	60	154	124	150	0.68	34.26	6.29	60	2.5	173.5	1.026	0.95
	29	75	151.5	122.5	148.5	0.45	30.21	4.42	60	1.0	100.0	0.981	1.08
	30	120	153.9	120.2	146.2	0.7	30.21	4.42	60	1.0	86.5	0.994	0.99
	31	135	154.5	124	150	1.19	32.89	5.50	60	2.0	119.5	1.077	1.02
	32	120	154	120.2	146.2	1.23	32.89	5.50	60	2.0	132.5	1.035	1.10
	35	135	155.1	122.5	148.5	1.48	31.35	4.91	60	1.5	124.5	1.066	0.91
	36	60	154.8	122	148	0.44	31.35	4.91	60	1.5	123.5	0.992	1.02
	37	135	153.8	123.1	149.1	1.49	32.08	5.72	60	2.0	140.0	1.022	0.94
	38	110	152.2	124	150	0.44	32.08	5.72	60	2.0	74.0	1.102	1.05
	39	110	153.5	124	150	1.2	31.35	5.64	60	2.25	144.5	1.044	1.08
	40	125	155.5	122.8	148.8	1.47	31.35	5.64	60	2.25	142.0	1.070	0.99
	44	135	153.8	122.6	148.6	1.21	28.67	4.85	60	1.50	109.5	1.085	1.00
	45	135	153	122.3	148.3	1.5	28.19	4.37	60	1.0	120.0	0.988	0.94
	46	75	154.5	92	146	0.45	28.19	4.37	60	1.0	74.5	1.031	0.98
	48	80	155.5	93.2	148.2	0.68	28.92	5.16	60	2.0	100.0	1.045	0.98
	49	80	154.1	122.1	148.2	1	30.46	5.81	60	2.5	164.5	1.031	1.03

Continued Table A.2

References	Corbel Number	a (mm)	b(mm)	d (mm)	h (mm)	ρ (%)	f_c (MPa)	f_t (MPa)	l/d_f	v_f(%)	V_{exp} (kN)	$V_{(Eq.7)}/V_{exp}$	V_{SVM}/V_{exp}
Fattuhi (1994b)	51	110	153.4	132.3	148.3	1.00	31.27	5.83	60	2.0	130.5	1.059	0.94
	52	110	152.2	94	150	1.00	31.27	5.83	60	2.0	99.0	1.012	0.95
	53	135	153.6	133.6	149.6	1.48	33.29	5.68	60	2.0	144.5	1.044	1.08
	54	135	151.7	93.8	149.8	1.49	33.29	5.68	60	2.0	101.5	1.079	0.97
	55	75	153.7	135.3	149.3	0.44	29.89	4.06	60	1.0	104.0	1.023	1.02
	56	75	152.9	115.8	149.8	0.44	29.89	4.06	60	1.0	95.5	0.978	0.94
	57	80	152.2	135.1	150.1	0.69	31.43	5.92	60	2.0	138.5	1.035	1.01
	58	80	152.8	113.3	148.3	0.69	31.43	5.92	60	2.0	121.5	1.050	1.00
	59	135	153	114	150	0.99	29.32	5.37	60	2.0	97.5	1.004	1.02
	60	110	152.8	112.6	148.6	1.49	29.32	5.37	60	2.0	142.0	1.001	1.07
	61	60	152.6	95	149	0.44	29.40	4.82	60	1.5	98.5	1.021	1.05
	62	135	153	114.1	150.1	1.20	29.40	4.82	60	1.5	109.5	1.008	1.00
	63	80	153	94	150	0.68	30.94	5.94	60	2.5	101.8	1.092	1.05
	64	60	152.6	92.5	147.5	1.00	30.94	5.94	60	2.5	170.0	1.001	0.95
	75	75	154.3	125.9	149.9	0.44	25.11	4.05	60	1.0	94.8	1.017	1.05
	76	75	154.5	94.8	148.8	0.44	25.11	4.05	60	1.0	73.5	1.029	1.01
	77	110	153.1	122.3	148.3	1.00	26.89	4.96	60	1.5	114.5	1.050	0.93
	78	135	153.1	121.7	147.7	1.50	26.89	4.96	60	1.5	120.0	1.019	1.04
	79	135	153.2	123.4	149.4	1.48	27.38	5.26	60	2.0	128.0	1.020	1.07
	80	110	154	122.1	148.1	1.00	27.38	5.26	60	2.0	120.8	1.032	0.99
	81	135	153.6	121.6	147.6	1.22	28.67	5.04	60	2.0	110.8	1.103	0.99
	82	110	154	92	148	1.00	28.67	5.04	60	2.0	98.0	0.986	1.06
	83	135	150.4	111.9	147.9	1.53	28.27	4.96	60	1.5	115.3	1.029	1.06
	84	135	152.4	91.7	147.7	1.51	28.27	4.96	60	1.5	94.0	1.056	0.95
	85	110	154.2	112.5	148.5	0.99	28.43	5.17	60	2.0	123.3	0.942	1.02
	86	135	153.2	113.9	149.9	1.48	28.43	5.17	60	2.0	115.5	1.089	0.96
	87	60	152.9	93.5	148.5	0.69	29.32	6.01	60	2.5	139.8	1.014	0.98
	88	80	153.1	93.1	149.1	1.00	29.32	6.01	60	2.5	138.8	0.952	0.96
Mean												1.007	1.01
Std. Dev.												0.058	0.05
CoV												0.058	0.046
RMSE												6.731	4.735
R^2												**0.949**	**0.975**

Table A.3. Material and geometric properties of analyzed RC haunched beams and comparison models

Source	Specimen	B (cm)	d_s (cm)	α^o	ρ_s*	ρ_v*	f_c (MPa)	f_y (MPa)	V_{exp} (kN)	V_{pred} (kN)	V_{pred}/V_{exp}
Debaiky and Elniema (1982)	A2	12	11	9.46	0.00198	0.03	20	461	58	58.02	1.00
	B3	12	11	9.46	0.00198	0.03	18.6	461	65.5	65.48	1.00
	B4	12	11	9.46	0.00198	0.03	21	461	101.5	101.51	1.00
	C2	12	11	9.46	0.00198	0.03	28.2	461	72	71.98	1.00
	D3	12	11	9.46	0.00396	0.03	29.6	461	69	68.98	1.00
	D4	12	11	9.46	0.00419	0.03	27.5	461	58.5	58.49	1.00
	F3	12	11	9.46	0.00235	0.019	21.5	461	44	44.01	1.00
	F4	12	11	9.46	0.00235	0.024	21	461	45.5	45.51	1.00
	A3	12	18.5	4.76	0.00198	0.03	17.8	461	78.5	78.53	1.00
	C3	12	18.5	4.76	0.00198	0.03	27.8	461	52	51.96	1.00
	A4	12	33.5	–4.76	0.00198	0.03	22	461	51.3	51.30	1.00
	C5	12	33.5	–4.76	0.00198	0.03	31.4	461	57.5	57.50	1.00
	E2	12	33.5	–4.76	0.00314	0.032	33.5	461	75	75.04	1.00
	A5	12	41	–9.46	0.00198	0.03	22.5	461	57	56.99	1.00
	B5	12	41	–9.46	0.00198	0.03	20.6	461	78.5	78.48	1.00
	C4	12	41	–9.46	0.00198	0.03	31.1	461	61	61.02	1.00
	D5	12	41	–9.46	0.00396	0.03	28.9	461	65	65.02	1.00
	D6	12	41	–9.46	0.00419	0.03	32.2	461	75	75.02	1.00
	E1	12	41	–9.46	0.00314	0.032	34.8	461	95	94.94	1.00
	F1	12	41	–9.46	0.00235	0.019	21.1	461	67	66.96	1.00
	F2	12	41	–9.46	0.00235	0.024	20.8	461	70.5	70.52	1.00
Tena-Colunga et al. (2008)	TASC1-0	22	41	–3.07	–	0.0263	32.1	412	67.5	67.51	1.00
	TASC2-0	22	41	–6.12	–	0.0308	29.5	412	60	59.99	1.00
	TASC3-0	22	41	–9.13	–	0.0372	23.6	412	37.5	37.53	1.00
	TASC4-0	22	41	–12.1	–	0.047	28.1	412	30	30.02	1.00
	TASC1-1	22	41	–3.07	0.0025	0.0263	26.9	412	200	200.01	1.00
	TASC2-1	22	41	–6.12	0.0025	0.0308	29.2	412	170	170.01	1.00
	TASC3-1	22	41	–9.13	0.0025	0.0372	28.8	412	120	119.99	1.00
	TASC4-1	22	41	–12.1	0.0025	0.047	21.1	412	80	79.98	1.00
Stefanou (1983)	B1-Ib	10	10	13.39	–	0.02	19.9	361	25	24.99	1.00
	B2-Ia	10	15	8.13	–	0.013	19.9	361	26.5	26.46	1.00
	B2-Ib	10	15	8.13	–	0.02	19.9	361	30	26.46	0.88
	B3-Ia	10	10	13.39	–	0.013	15.7	361	27.5	25.02	0.91
	B3-Ib	10	10	13.39	–	0.02	15.7	361	25	25.02	1.00
	B4-Ia	10	15	8.13	–	0.013	15.7	361	26.5	32.51	1.23
	B4-Ib	10	15	8.13	–	0.02	15.7	361	32.5	32.51	1.00
	B5- Ias	10	10	13.39	0.0032	0.013	19.9	361	22.5	27.00	1.20
	B5-Ibs	10	10	13.39	0.0032	0.02	19.9	361	27	27.00	1.00
	B6- Ias	10	15	8.13	0.0032	0.013	19.9	361	29	29.06	1.00
	B6-Ibs	10	15	8.13	0.0032	0.02	19.9	361	37.75	29.06	0.77
	B7- Ias	10	10	13.39	0.0032	0.013	15.7	361	29	29.00	1.00
	B8- Ias	10	15	8.13	0.0032	0.013	15.7	361	27.5	27.52	1.00
Nghiep (2011)	2L	20	20	3.95	–	0.0157	49.4	550	75	75.00	1.00
	3L	20	15	5.91	–	0.0157	50.2	550	66.5	66.52	1.00
	2K	20	24.3	3.95	–	0.0157	54	550	83.5	83.49	1.00
	3K	20	20	6.71	–	0.0157	54	550	79.5	79.50	1.00
	4K	20	15	10.01	–	0.0157	54	550	85	84.99	1.00
Mean											1.00
Std. Dev.											0.06
CoV											0.06
MSE											3.2
R^2											0.997