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A FAST AND ACCURATE METHOD TO PREDICT RELIABILITY OF PROJECT COMPLETION TIME

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Abstract. To meet the target completion time of a construction project is one of the most important performance indicators of project management. This paper proposes a fast and accurate method for evaluating the reliability of project completion time in large construction projects, using reliability theory. The proposed method is developed to overcome the limitations of existing methods, including the inaccuracy of the program evaluation and review technique and the long computational time of the narrow reliability bounds method. The proposed method is established in three main parts: (i) calculating the statistics of paths duration in the network; (ii) truncating insignificant paths of the network; and (iii) proposing an innovative solution to accurate estimate for reliability of project completion time. The effectiveness of the proposed method is evaluated using an example project. It is found that the results of the proposed method on the reliability of completion time are accurate. It is also found that the proposed method significantly reduces the number of analysed network paths and the computational effort. The method proposed here can serve as a fast and accurate tool for project managers and project planners in project planning, re-planning, and project control phases.

Keywords: project completion time, variability, reliability method, project planning and control.

Introduction

The critical path method (CPM) has been popularly used in the construction industry to estimate project completion time, or duration, for generations (Nasir *et al.* 2003; Mo *et al.* 2008; Okmen, Oztas 2008; Jun, El-Rayes 2011). This may largely be due to its simplicity since CPM assumes that the durations of project activities are deterministic. CPM uses forward-pass and backward-pass algorithms to calculate which sequence of activities has the least float (Lee, Arditi 2006).

Project activities, however, generally have variability associated with them, leading to uncertainty in the planning formulation. The variability in estimating of an activity's duration may arise from various features, primarily the work itself and the estimator having insufficient data to characterise the work exactly (Carmichael 2006). This variability has led to the consideration of probability methods, both analytically such as PERT (Program Evaluation and Review Technique) (Halpin, Riggs 1992; Ahuja *et al.* 1994; Kerzner 2009) and NRB (Narrow Reliability Bounds) (Ditlevsen 1979; Schuëller, Stix 1987; Melchers 1999; Rackwitz 2001), and numerically, such as Monte Carlo Simulation (MCS) (Lu, AbouRizk 2000; Lee, Arditi 2006) and Simplified Monte Carlo Simulation (SMCS) (Diaz, Hadipriono 1992).

Simulation methods are widely used as a practical technique to incorporate variability in construction projects (Lu, AbouRizk 2000; Lee, Arditi 2006). However, simulation approaches require large samples for large-scale projects in which each simulation requires the scheduling of all the project activities, including the forward path analysis of the CPM to calculate the project completion time (Ang *et al.* 1975; Lu, AbouRizk 2000; Guo *et al.* 2001; Zammori *et al.* 2009).

Although simulation methods are well established in construction, it is still desirable to formulate and solve the problem of the reliability of project completion time by analytical methods, even with simplifications (Li, Melchers 1993). This is largely because analytical approaches assist practitioners in greater understanding of the problem nature by evaluating the parameters involved in the problem (Li, Melchers 1993). In evaluating the reliability of a project completion time, however, analytical methods, such as PERT and NRB, are not widely used. This might be due to some limitations that exist in such analytical methods. PERT, developed by the US Navy in 1958, introduces uncertainty into the estimates for activity durations (Kerzner 2009; Jun, El-Rayes 2011). In PERT, calculations are performed on an expected activity duration together with an associated measure of the vari-

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ability of this duration. However, PERT only considers the critical path of a network and it neglects the impact of correlation between the network paths (Jun, El-Rayes 2011). This may lead to unrealistic results (Ahuja *et al.* 1994; Halpin, Riggs 1992). Compared with PERT, NRB takes all the paths into consideration, which provides more realistic results. However, NRB requires a large number of calculations (Ditlevsen 1979), which might not be ideal for practical use by construction managers. Other methods have also been proposed, such as approximation methods (Ang *et al.* 1975; Gong, Hugsted 1993; Guo *et al.* 2001), multivariate methods (Anklesaria, Drezner 1986) and fast and accurate multivariate methods (Jun, El-Rayes 2011). All these can be referred to in literature.

Despite significant contributions of the above mentioned studies to the reliability of project completion time, a comprehensive survey of research literature in the area of construction management as shown in Table 1, suggests that there are few or no studies that have focused on developing an accurate and fast analytical method to determine the reliability of project completion time. This paper addresses this shortfall. The paper proposes a fast and accurate analytical method, an original and practical one, for evaluating the reliability of project completion time of large construction projects. The benefit of developing such a method to construction managers is that the proposed method significantly improves the efficiency, accuracy, practicality and confidence in evaluating the reliability of project completion time for large-scale construction projects. The proposed method is designed to overcome the limitations of existing analytical reliability methods including: the inaccuracy of the program evaluation and review technique (PERT) and the long computational time of the narrow reliability bounds (NRB) method. The paper addresses a real life problem in construction project management by proposing an accurate and fast analytical solution to the reliability problem of project completion time, thereby, contributing to current knowledge and practices in project planning and control. The solution follows an ordered argument and can be easily usable by practitioners.

In order to achieve this, the reliability problem of project completion time is formulated. Then a fast and accurate analytical method for estimating the reliability of project completion time is developed. The method is

Table 1. Methods for the reliability of project completion time in construction

Method	Methodology	Advantages	Limitations	Reference
Deterministic	The critical path method (CPM)	Simple	Incapable of considering the impact of various construction risks and uncertainties	Nasir *et al.* (2003); Mo *et al.* (2008); Okmen and Oztas (2008); Jun and El-Rayes (2011)
Analytical (probabilistic)	Program evaluation and review technique (PERT)	Probabilistic scheduling capabilities; simple and practical	Ignore all subcritical paths; produce unrealistic results	Halpin and Riggs (1992); Ahuja *et al.* (1994); Kerzner (2009)
	Narrow reliability bounds (NRB)	Consider all network paths; provide more realistic results	Require a large number of calculations	Ditlevsen (1979)
Numerical (probabilistic)	Monte Carlo simulation (MCS)	Provide the most accurate results	Require large computation load for large-scale projects	Halpin and Riggs (1992); Diaz and Hadipriono (1993); Lu and AbouRizk (2000); Lee and Arditi (2006)
	Simplified Monte Carlo simulation (SMCS)	Consider all network paths	The use of a computer is recommend	Diaz and Hadipriono (1992)
Approximation (probabilistic)	Probabilistic network evaluation technique (PNET)	Consider all network paths	Produce optimistic results	Ang *et al.* (1975)
	Multivariate normal distribution.	Consider all network paths	The durations of all activities are assumed to be normally distributed	Sculli and Shum (1991)
	Back-forward uncertainty-estimation (BFUE)	Consider all network paths	The real relationship between the merging paths might be complicated.	Gong and Hugsted (1993)
	The modified stochastic assignment model (MSAM)	Consider all network paths	When the variance is large compared to the mean value is not applicable	Guo *et al.* (2001)
Multivariate (probabilistic)	Multivariate normal distribution	Consider all network paths	Applicable for small networks	Anklesaria and Drezner (1986)
	Fast and accurate risk evaluation (FARE)	Reduce the number of calculations	The use of a computer is recommend	Jun and El-Rayes (2011)

developed in three main parts: (i) calculating the statistics of paths duration in the network; (ii) truncating insignificant paths of the network; and (iii) proposing an innovative solution to accurate estimate for the reliability of project completion time. A worked example of a real construction project is presented to demonstrate the application of the developed method and to show its capabilities in providing fast and accurate solution to determining the reliability of project completion time.

1. Problem formulation

A project network, typically, consists of many activities, some of which can proceed in parallel, while others can only proceed after certain preceding activities have been completed. In reality, the completion time or duration for each activity [Note, the terms "completion time" and "duration" in this paper are interchangeable] is not certain because there are so many factors, such as weather condition, materials delay, productivity and site condition that may affect the activity duration (Yang 2005; Yang *et al.* 2014). As such, the completion time of the project should be treated as a random variable. In addition, such factors have a significant effect on the project final cost. Whilst this paper focuses on project completion time, it is acknowledged that project completion time also affects the project cost.

Typically, the project completion time is the completion time of the longest path of the project network, called critical path and denoted by T_C in this paper.

In evaluating the reliability of completion time for the critical path a criterion should be established. In reliability theory, this criterion can be expressed by a limit state function, as follows (Li, Melchers 2005):

$$G(T_C, T_L) = T_C - T_L , \tag{1}$$

where T_L is an acceptable limit for the project completion time, for example a target completion time (duration). From Eqn (1), the reliability of completion time, denoted by *R*, for the critical path can be determined by (Li, Melchers 2005):

$$R = P\left[G\left(T_C, T_L\right) < 0\right] = P\left[T_C < T_L\right], \tag{2}$$

where *P*[ ] denotes the probability of an event. Following reliability theory (for example, Melchers 1999), T_C and T_L are treated as random variables with f_{T_C} (*u*) and f_{T_L} (*v*) as their probability density functions, respectively, and *u* and *v* are random variables. It follows that when T_C and T_L are random variables Eqn (2) can be expressed as:

$$R = \iint\limits_D f_{T_C T_L}\left(u, v\right) du dv , \tag{3}$$

where $f_{T_C T_L}\left(u, v\right)$ is the joint probability density function of T_C and T_L; and *D* is the domain that represents $T_C < T_L$.

When T_C and T_L are independent, Eqn (3) can be obtained by (Melchers 1999):

$$R = \int_{-\infty}^{+\infty} F_{T_L}\left(z\right) f_{T_C}\left(z\right) dz , \tag{4}$$

where *F*( ) is a cumulative distribution function; *f*( ) is a probability density function; and *z* is a random variable.

As a special case when both the completion time of the critical path and the acceptable limit are normal random variables; that is $T_C \sim N(\mu_C, \sigma_C^2)$, and $T_L \sim N(\mu_L, \sigma_L^2)$, the analytical solution of Eqn (4) is possible and can be obtained by (Melchers 1999):

$$\mathrm{R} = \Phi\left(-\frac{\mu_\mathrm{C} - \mu_\mathrm{L}}{\sqrt{\sigma_\mathrm{C}^2 + \sigma_\mathrm{L}^2}}\right), \tag{5}$$

where Φ( ) is the standard normal distribution function (zero mean and unit variance).

In the case that T_L is the project target completion time and has a deterministic value, *t*, Eqn (5) can be expressed as:

$$R = \Phi\left(-\beta_C\right), \tag{5a}$$

where β_C is referred to as completion index in this paper and is defined as:

$$\beta_C = \frac{\mu_C - t}{\sigma_C} . \tag{5b}$$

Equation (5a) is used by PERT for the longest path in a network to estimate the reliability of project completion (Kerzner 2009; Jun, El-Rayes 2011). PERT determines the longest path deterministically by using the critical path method. Despite the simplicity of PERT, it is rather exceptional to evaluate the reliability of the project completion by considering only one path and neglecting the influence of correlation between the network paths. Accordingly, PERT produces inaccurate results, often optimistic, that is uncorrelated, results (Halpin, Riggs 1992; Ahuja *et al.* 1994; Jun, El-Rayes 2011).

In order to consider the effect of all paths on the reliability of the project completion time, the network needs to be modelled as a series system (Melchers 1999). By a system approach, the reliability of the project completion time can be expressed by:

$$R = 1 - p_f , \tag{6}$$

where p_f is the probability of the project completion time to be longer than a target time.

For a series system, the project completion time being longer than the target time is the union of the completion times of all possible paths being finished longer than the target time:

$$p_f = P\left(F_1 \cup F_2 \cup ... \cup F_k\right), \tag{7}$$

where F_j corresponds to the event that path *j*'s completion time is longer than a target time; the notation ∪ denotes the union of events; and *k* is the number of the paths in the project network.

Following reliability theory, p_f can be determined by (Melchers 1999):

$$p_f = \int\limits_{D \in x} ... \int f_x\left(x\right) dx , \tag{8}$$

where *x* represents the vector of all the basic random variables affecting the completion time of the project, such as weather condition, productivity and so on.

In series systems, for most practical cases, Eqn (8) is rather complicated to evaluate (Schuëller, Stix 1987; Rackwitz 2001). Rather than attempting the direct integration of Eqn (8) an alternative approach is to develop upper and lower bounds for p_f. Following probability theory, Eqn (7) can be written as (Melchers 1999):

$$p_f = \sum_{i=1}^{k} P(F_i) - \sum_{i=1}^{k}\sum_{i<j}^{k} P(F_i \cap F_j) + \sum_{i=1}^{k}\sum_{i<j}^{k}\sum_{j<\vartheta}^{k} P(F_i \cap F_j \cap F_\vartheta) - \ldots, \tag{9}$$

where the notation $\cap$ refers to intersection of events; and $P(F_i \cap F_j)$ is the probability of the intersection of paths i and j completion times being longer than a target time.

It follows directly from Eqn (9) that, if $P(F_i) << 1$, then the terms $P(F_i \cap F_j)$, $P(F_i \cap F_j \cap F_\vartheta)$ and etc., are negligible. If only the terms $P(F_i)$ in Eqn (9) are retained, it can be shown that upper and lower bounds on p_f, referred to as first-order series bounds, can be obtained by (Grimmelt, Schuëller 1982–1983; Ramachandran 1984):

$$\max_{i=1}^{k}\left[P(F_i)\right] \le p_f \le \sum_{i=1}^{k} P(F_i). \tag{10}$$

Unfortunately, for many practical cases the series bounds provided by Eqn (10) are too wide; that is inaccurate to be meaningful (Grimmelt, Schuëller 1982–1983). To overcome this problem of inaccuracy due to wide bounds, second-order series bounds, also known as narrow reliability bounds (NRB), were proposed which retains terms, such as $P(F_i \cap F_j)$ in Eqn (9) (Melchers 1999). Following the NRB method, upper and lower bounds on p_f can be obtained by (Ditlevsen 1979):

$$p_{f_{LB}} = P(F_1) + \sum_{i=2}^{k} \mathrm{Max}\left[0, P(F_i) - \sum_{j=1}^{i-1} P(F_i \cap F_j)\right]; \tag{11a}$$

$$p_{f_{UB}} = P(F_1) + \sum_{i=2}^{k} \mathrm{Max}\left[0, P(F_i) - \sum_{j=1}^{i-1} P(F_i \cap F_j)\right], \tag{11b}$$

where the ordering of the network paths from 1 to k is based on their decreasing significance; that is $P(F_1) > P(F_2) > \ldots > P(F_k)$.

Using Eqn (6) an upper and a lower bound for R can be calculated by:

$$R_{UB} = 1 - \left\{P(F_1) + \sum_{i=2}^{k} \mathrm{Max}\left[0, P(F_i) - \sum_{j=1}^{i-1} P(F_i \cap F_j)\right]\right\}; \tag{12a}$$

$$R_{LB} = 1 - \left\{P(F_1) + \sum_{i=2}^{k}\left[P(F_i) - \mathop{\mathrm{Max}}_{j<i} P(F_i \cap F_j)\right]\right\}, \tag{12b}$$

where R_{UB} and R_{LB} are upper and lower bounds of the reliability of project completion time, respectively.

The lower and upper bounds of $P(F_i \cap F_j)$, which are used in Eqns (12a) and (12b) can be obtained by (Ditlevsen 1979):

$$P_{LB}(F_i \cap F_j) = \mathrm{Max}\left[\Phi(\beta_{C_i})\Phi\left(\frac{\beta_{C_j} - \rho_{ij}\beta_{C_i}}{\sqrt{1-\rho_{ij}^2}}\right), \Phi(\beta_{\beta_{C_j}})\Phi\left(\frac{\beta_{C_i} - \rho_{ij}\beta_{C_j}}{\sqrt{1-\rho_{ij}^2}}\right)\right]; \tag{13a}$$

$$P_{UB}(F_i \cap F_j) = \Phi(\beta_{C_i})\Phi\left(\frac{\beta_{C_j} - \rho_{ij}\beta_{C_i}}{\sqrt{1-\rho_{ij}^2}}\right) + \Phi(\beta_{C_j})\Phi\left(\frac{\beta_{C_i} - \rho_{ij}\beta_{C_j}}{\sqrt{1-\rho_{ij}^2}}\right), \tag{13b}$$

where β_{C_i} is a completion index obtained in Eqn (5b) by letting $C = i$; and ρ_{ij} is the correlation coefficient between paths i and j.

It has been shown that for a range of distribution types and a range of variances the results of NRB are very accurate and close to the simulation results (Grimmelt, Chueller 1982–1983; Diaz, Hadipriono 1993). Unfortunately, the application of NRB to evaluating the reliability of project completion time is time consuming when the size of the network is large. This is because in a large-scale project with many paths, Eqns (13a) and (13b) need to be calculated for any pair of paths which results in a large number of calculations. To overcome this large computational problem a novel method named "Fast and Accurate Reliability Bounds" (FARB) is developed as presented in the next section.

2. Fast and accurate reliability bounds

Fast and Accurate Reliability Bounds (FARB) can provide a novel analytical solution to determining the reliability of completion time of large-scale construction projects. The method is designed to overcome the limitations of existing analytical reliability methods, including: the inaccuracy limitation of PERT attributable to neglecting the correlation between the network paths by incorporating a multipath method; and the long computational time of NRB. The FARB solution consists of three main parts: (i) calculating the statistics of paths duration; (ii) truncating insignificant paths; and (iii) developing an innovative solution to provide a fast and accurate estimate for the reliability of completing time in large-scale construction projects.

2.1. Calculating the statistics of paths completion time

In order to calculate the reliability of completion time for each path S in a project network, its associated mean (μ_S) and variance (σ_S^2) should be obtained. These parameters are calculated in FARB based on probabilistic theory and can be determined by:

$$\mu_S = \sum_{j=1}^{n_S} \mu_{a_j}; \tag{14}$$

$$\sigma_S^2 = \sum_{j=1}^{n_S} \sigma_{a_j}^2 + \sum_{j=1}^{n_S} \sum_{q=j+1}^{n_S} \sigma_{a_j} \sigma_{a_q} \eta_{a_{jq}}, \quad (15)$$

where μ_{a_j} is the mean duration of activity j, located on path S; n_S is the number of activities on path S; $\sigma_{a_j}^2$ is the variance of the duration of activity j; and $\eta_{a_{jq}}$ is the correlation between activities j and q.

In practice, it is most likely that all activities are statistically independent; that is $\eta_a = 0$, and accordingly, each path S duration variance can be obtained by:

$$\sigma_S^2 = \sum_{j=1}^{n_S} \sigma_{a_j}^2. \quad (16)$$

If sufficient activities are included in a network path, and the activities are not correlated, the central limit theory dictates that each path S duration can be approximated to a normal distribution with mean and variance obtained in Eqns (14) and (16), respectively. Noting that, considerable activities have to be added in a path before the central limit theorem becomes applicable. Five to ten activities seem to be a minimum number required (Carmichael 2006).

Equations (14) and (16) require the statistics of activity durations. As mentioned before, the duration of each activity j is a random variable, depending on basic random variables, such as weather, productivity, site condition, crew motivation and so on. In practice, the probabilistic information of the basic random variables is not easy to obtain. To overcome this difficulty, the approach of PERT is widely used to calculate the mean and variance of each activity j duration based on three descriptors: an optimistic activity duration, denoted t_o; a pessimistic activity duration, denoted t_p; and a most likely activity duration, denoted t_m. All three descriptors are, accordingly, the planner's estimates of an activity's duration reflecting the nature of the activity or the planer's own uncertainty of the activity duration. Notably, the variability given to the activity duration estimate should not cover infrequent and unusual circumstances, such as, industry disputes, accidents, changes in work methods, alterations in the resources used, or similar (Carmichael 2006). Following PERT, each activity j mean and variance can be, respectively, obtained by:

$$m_{a_j} = \frac{t_o + 4t_m + t_p}{6}; \quad (17)$$

$$\sigma_{a_j}^2 = \left[\frac{t_p - t_o}{6}\right]^2. \quad (18)$$

2.2. Truncating insignificant paths

In order to calculate the reliability of completion time for a given project network, Eqns (12a) to (13b) are required to be calculated by considering the correlation between all paths in the network. This leads to a large number of calculations for large-scale projects. For this purpose, a scheme is developed to truncate the number of paths.

The truncation of the paths numbers in a project network is based on the principle proposed by Ang *et al.* (1975):

- the paths in a network with high mean durations and high variances have the greater effect on the reliability of project completion duration;
- if durations on several paths are highly correlated, these paths are replaced by a single representative path which has the highest variance among each set of correlated paths; and
- paths with a low correlation coefficient are assumed independent and are grouped as other representative paths.

Following Ang *et al.* (1975) and Carmichael (2006), the correlation coefficients between any two paths i and j are calculated, assuming all activities are statistically independent, by:

$$\rho_{ij} = \frac{\sum_m \sigma_{a_m}^2}{\sigma_i \sigma_j}, \quad (19)$$

where m includes all activities common to paths i and j.

For every pair of paths i and j, the correlation coefficient, ρ_{ij}, is compared with a correlation coefficient, ρ_0. When $\rho_{ij} < \rho_0$, paths i and j are treated as independent paths. Ang *et al.* (1975) recommend that a value of 0.5 for ρ_0 is appropriate for construction networks.

2.3. Innovative solution

This part develops an innovative solution to estimating the reliability of project completion time of large-scale construction projects in which their scheduled network may include tens of thousands of network paths. The development is based on the principles presented in parts (i) and (ii), and the reliability theory, discussed earlier.

Considering paths i and j are two representative and independent paths. Following probability theory, for any pair of independent events F_i and F_j, the probability that events F_i and F_j both occur is:

$$P\left(F_i \cap F_j\right) = P\left(F_i\right) P\left(F_j\right). \quad (20)$$

Substituting Eqn (20) into Eqns (12a) and (12b), for m representative and independent paths, upper and lower bounds for the reliability of the project completion time being shorter than t are, respectively, obtained by:

$$R_{UB} = 1 - \left\{\Phi\left(\beta_{C_1}\right) + \sum_{i=2}^{k} \text{Max}\left[0, \Phi\left(\beta_{C_i}\right) - \sum_{j=1}^{i-1} \Phi\left(\beta_{C_i}\right)\Phi\left(\beta_{C_j}\right)\right]\right\}; \quad (21a)$$

$$R_{LB} = 1 - \left\{\Phi\left(\beta_{C_1}\right) + \sum_{i=2}^{k}\left[\Phi\left(\beta_{C_i}\right) - \underset{j<i}{\text{Max}}\left\{\Phi\left(\beta_{C_i}\right)\Phi\left(\beta_{C_j}\right)\right\}\right]\right\}, \quad (21b)$$

where β_{C_i} is a completion index for path i obtained in Eqn (5b).

Notably, the results from the proposed method, FARB, should be accurate as Eqns (21a) and (21b) are derived based on NRB, which has been proved an accurate method (Grimmelt, Schuëller 1982–1983; Diaz, Hadipriono 1993). Also FARB is a fast method as Eqns (21a) and (21b) are calculated only for the represented paths of a network without any need for calculating Eqns (13a) and (13b) for any pairs of the paths in a network. Accordingly, FARB should significantly improve the efficiency, accuracy, practicality and confidence in predicting project completion time of large-scale construction projects. The next section presents the application of the proposed method.

3. Worked example

An example construction project is analysed to demonstrate the use of the developed FARB method and to show its capabilities in providing fast and accurate estimate for the reliability of completion time in construction projects. The project, which was first presented by Brook *et al.* (1967), concerns the construction of a road pavement. The project involves the paving of 2.2 miles of road and the construction of appurtenant drainage structures, excavation to grade, placement of macadam shoulders, erection of guardrails, and landscaping. For more details about the project activities, and their duration means and variances see Guo *et al.* (2001) due to space limitation of the paper.

Figure 1 illustrates the project network, Table 2 lists the duration statistics of individual activities, and Table 3 gives all of the nine paths of activity network for the exampl project. Table 4 shows means and variances of the durations of all paths. It also illustrates the correlation coefficients between the paths. The mean (μ) and variance (σ^2) of the duration of each path were, respectively, calculated by Eqns (14) and (16) and the correlation coefficients between any two paths were obtained by Eqn (19) for the example project network. In Table 4, the paths were sorted in a descending order on the basis of their duration's variance. This is to remove highly correlated paths from the analysis and replace them with representative paths that have the highest variance among each set of correlated paths. Notably, Ang *et al.* (1975) suggest sorting the paths in a descending order on the basis of their duration's mean. Their approach, however, does not guarantee the selection of representative paths that have lower reliability of completion time (Jun, El-Rayes 2011). In practice, project managers should pay more attention to controlling any factors (risks) that affect both the path with the longest duration, i.e., the critical path, and the path with the highest variance. This is because such paths have the highest effect on the reliability of project completion time.

Following Ang *et al.* (1975), paths with a correlation coefficient, ρ, greater than $\rho_0 = 0.5$ were considered to be dependent. That is, paths 2 to 5 are represented by path 1, and paths 7 to 9 are represented by path 6. The FARB method considers paths 1 and 6 as the network representative paths, and it calculates the reliability of the

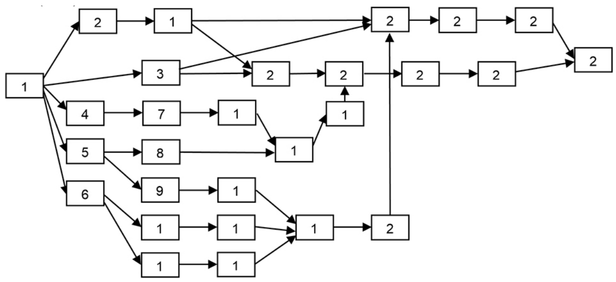

Fig. 1. Activity network of the exemplar project

Table 2. Duration statistics of activities

Activity	t_o (days)	t_m (days)	t_p (days)	m_a (days)	σ_a^2 (days)^2
1	0	0	0	0	0.0
2	3	1	6	2	0.3
3	2	5	8	5	1.0
4	2	6	11	6	2.3
5	2	3	5	3	0.3
6	1	4	25	7	16.0
7	4	10	16	10	4.0
8	2	2	8	3	1.0
9	5	6	14	7	2.3
10	1	4	13	5	4.0
11	3	1	12	3	2.3
12	3	9	15	9	4.0
13	3	4	12	5	2.3
14	2	3	5	3	0.3
15	2	6	29	9	20.3
16	2	5	14	6	4.0
17	3	1	6	2	0.3
18	2	7	12	7	3.0
19	1	4	13	5	4.0
20	4	10	16	10	4.0
21	1	5	21	7	11.0
22	2	6	11	6	2.3
23	3	7	30	10	20.3
24	2	6	11	6	2.3
25	2	2	8	3	1.0
26	2	2	8	3	1.0
27	3	4	12	5	2.3
28	0	0	0	0	0.0

Table 3. Paths of activity network for the example project

Path	Activities in path
1	1, 6, 10, 15, 19, 21, 23, 24, 26, 27
2	1, 6, 11, 16, 19, 21, 23, 24, 26, 27
3	1, 5, 9, 14, 19, 21, 23, 24, 26, 27
4	1, 3, 23, 24, 26, 27
5	1, 2, 17, 23, 24, 26, 27
6	1, 4, 7, 12, 13, 18, 20, 22, 25, 27
7	1, 5, 8, 13, 18, 20, 22, 25, 27
8	1, 3, 28, 20, 22, 25, 27
9	1, 2, 17, 28, 20, 22, 25, 27

Table 4. Ordered paths and duration statistics

Path (i)	Mean duration (μ_i) (days)	Variance (σ_i^2) (Days)^2	Path correlations								
			ρ_{i1}	ρ_{i2}	ρ_{i3}	ρ_{i4}	ρ_{i5}	ρ_{i6}	ρ_{i7}	ρ_{i8}	ρ_{i9}
1	57	81.0	1								
2	52	62.88	0.79	1							
3	49	43.43	0.69	0.78	1						
4	29	26.73	0.55	0.63	0.76	1					
5	28	26.21	0.56	0.63	0.76	0.97	1				
6	61	25.0	0.05	0.06	0.07	0.09	0.09	1			
7	42	16.00	0.06	0.07	0.09	0.11	0.11	0.74	1		
8	29	10.5	0.08	0.09	0.11	0.19	0.69	0.59	0.73	1	
9	28	9.99	0.08	0.09	0.11	0.14	0.17	0.60	0.75	0.93	1

project completion time by considering these two representative paths. It should be noted that for the example project Eqns (13a) and (13b) are required to be calculated 36 times by the NRB method. In comparison, those Equations are not required to be calculated by FARB. Accordingly, FARB reduces the amount of calculations for large-scale projects significantly. Using Eqns (21a) and (21b), the reliability of project completion for different target durations, t, can be calculated by:

$$R_{UB} = 1 - \left\{ \Phi\left(\beta_{C_1}\right) + \text{Max}\left[0, \Phi\left(\beta_{C_2}\right) - \Phi\left(\beta_{C_1}\right)\Phi\left(\beta_{C_2}\right)\right]\right\}; \quad (22a)$$

$$R_{LB} = 1 - \left\{ \Phi\left(\beta_{C_1}\right) + \Phi\left(\beta_{C_2}\right) - \Phi\left(\beta_{C_1}\right)\Phi\left(\beta_{C_2}\right)\right\}. \quad (22b)$$

The calculation results show that the second term of the bracket in Eqn (22a) is positive. Accordingly, for this project $R_{UB} = R_{LB}$ which provides accurate value rather than bound. This is supported with the Diaz and Hadipriono (1993) study. They showed that the lower and upper bounds obtained with NRB are very close to each other. To compare the results of FARB with NRB, Eqns (12a) to (13b) were calculated by considering all paths of the exemplar project.

Figure 2 shows the reliability of project completion for the different target durations, t, produced by FARB and NRB. In this figure upper bound and lower bound are denoted by *UB* and *LB*, respectively.

Figure 2 demonstrates that the results for the reliability of project completion time obtained by FARB are in between those of upper and lower bounds of NRB. The maximum difference between the FARB results and those of NRB for lower bound is 0.06 and for upper bound is 0.15. This demonstrates that FARB produces results close to NRB.

To further evaluate the accuracy of the FARB method, Monte Carlo simulation was used to analyse the exemplar project. The maximum number of samples for MCS was set to be 10,000. Diaz and Hadipriono (1993) argue that 1000 interactions give a good result. The reliability of project completion time in MCS was calculated by:

$$R = \frac{n}{N}, \quad (23)$$

where N is the total number of samples conducted by MCS; and n is the total number of times that the project duration is shorter than the target duration, t, during simulation.

Figure 3 compares results of the reliability of project completion time produced by the FARB method and those of the Monte Carlo simulation method. Figure 3 demonstrates that FARB results are in good agreement with MCS results.

The differences between the results produced by Monte Carlo simulation and those of FARB are shown in Figure 4.

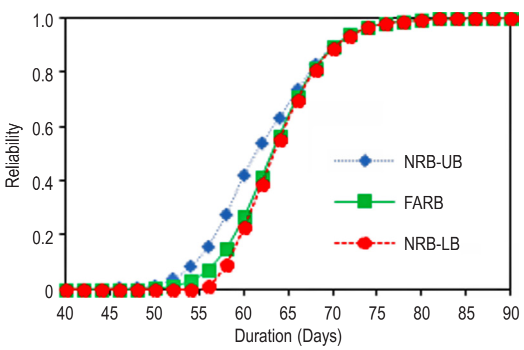

Fig. 2. Reliability of project completion time for different target durations

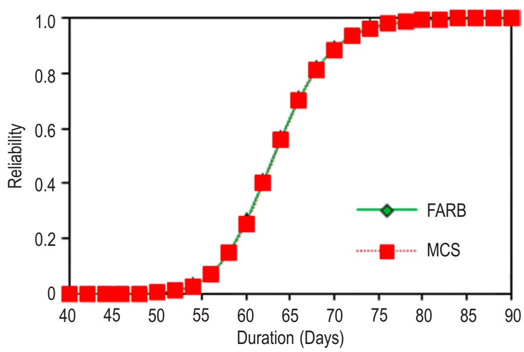

Fig. 3. Reliability of project completion time produced by FARB and Monte Carlo simulation

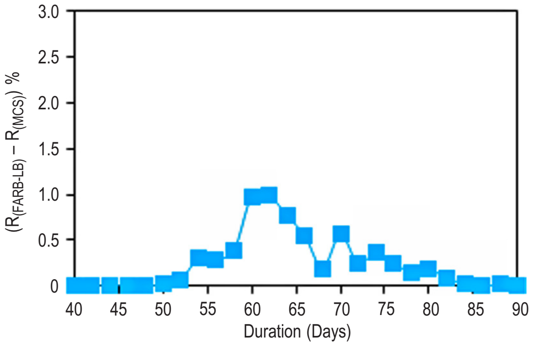

Fig. 4. Difference between reliability of project completion time produced by FARB and Monte Carlo simulation

Figure 4 illustrates that the maximum difference between the MCS results and those generated by FARB is less than 1.1%. To measure the significant of this, a t-test analysis was conducted. Using Figure 3 data, Table 5 details the result of the t-test. It needs to be noted that smaller t-values translate into larger p-values. So the smaller the t-value is the less likely the difference is significant. A critical t-value is the minimum t-value required to have $p < 0.05$. If the t-value is less than or equal to the critical t-value, then the difference between the two data sets are not statistically significant (Montgomery *et al.* 2012). In Table 5 the one-tailed test corresponds to the possibility of the difference between the two data sets in only one direction (positive or negative) while the two-tailed test corresponds to the possibility of the difference between the two data sets in both directions (positive and negative) (Montgomery *et al.* 2012).

Table 5. The t-test result for comparing results produced by FARB and Monte Carlo simulation

t-value	0.02
p-value one-tail	0.49
t-critical-value (one-tail)	1.68
p-value two-tail	0.98
t-critical-value (two-tail)	2.01

According to Table 5, the t-value is less than the t-critical value with a p-value higher than 0.05. This demonstrates that the difference between the reliability of the project completion time produced by MCS and FARB are not statistically significant (Montgomery *et al.* 2012). This suggests that FARB can produce very similar, if not exact, results to those generated by MCS. These accurate results were generated by significant reductions in the number of analysed network paths and the computational effort compared to NRB. Accordingly, FARB can significantly improve the efficiency, accuracy and practicality of utilizing analytical reliability techniques for project completion time in large-scale construction projects.

The above results were obtained by the consideration that paths with a correlation coefficient, ρ, greater than $\rho_0 = 0.5$ are dependent. To examine the effect of ρ_0 values on the results provided by FARB, considering paths with ρ, greater than $\rho_0 = 0.65$ are dependent. Accordingly, paths 2 through 3 are represented by path 1, path 5 is represented by path 4, path 7 by path 6 and path 9 by path 8. Using Eqns (20a) and (20b) upper and lower bounds for the reliability of project completion time for the different target durations, *t*, were calculated. Figure 5 shows the differences between upper and lower bounds of the reliability of project completion time produced by FARB with $\rho_0 = 0.65$. This figure demonstrates that the results for upper and lower bounds are so close to each other, and they can be considered as the same results.

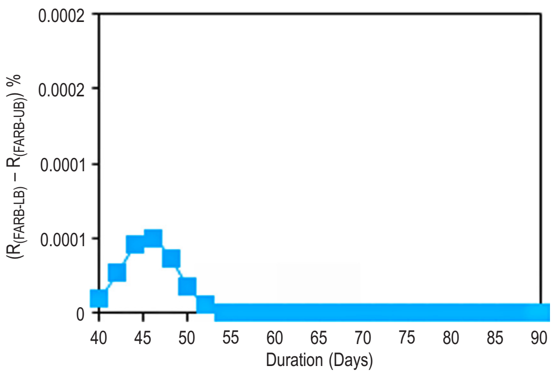

Fig. 5. Differences between upper and lower bounds of the reliability of project completion time with $\rho_0 = 0.65$

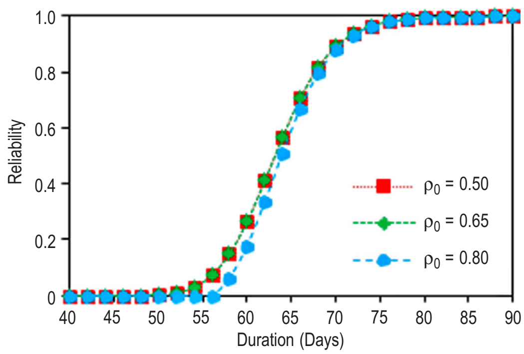

Fig. 6. Reliability of project completion time for different ρ_0 values

Figure 6 compares the reliability of project completion time produced by FARB for different ρ_0 values. This figure shows the results for $\rho_0 = 0.65$ and $\rho_0 = 0.50$ are close to each other. However, Figure 6 illustrates that for a high ρ_0 value, for example $\rho_0 = 0.8$, the results may become very conservative. As a lower value of ρ_0 requires smaller amount of calculation, a value of 0.5 for ρ_0 is appropriate for construction networks.

4. Further discussion

This paper proposes an accurate and fast analytical solution to the problem of reliability of completion time of construction project networks, thereby contributing to current knowledge and practice in project planning and control. The solution follows an ordered argument and is usable by practitioners. The paper will be of interest to academics and practitioners involved in the planning and control of construction projects. It provides an understanding of reliability analysis in projects networks; that is broader than existing treatments. Project managers and project planners, during planning and re-planning phases, might use an analytical approach to compare the Monte Carlo simulation results for the reliability of project completion; the proposed analytical method should assist planners in these phases. The proposed method should also significantly improve the efficiency, accuracy, practicality and confidence in evaluating the reliability of project completion time for large-scale construction projects. Although this paper largely focuses on the reliability of project completion time, the results provide guidance on analysing reliability problems in other areas of engineering with probabilistic natures.

It should be noted that problems associated with resource usage, resource smoothing and resource constrained scheduling, may arise as sub-problems of the total planning problem. Such sub-problems may or may not be significant problems depending on the degree to which resources are considered in the initial planning. At planning stages, resources are often treated in two broad approaches (Carmichael 2006): (a) resources are taken into consideration when establishing the durations of activities. No further action is required to be done by FARB; (b) resources are ignored (or resources are assumed to be available) when establishing the durations of activities. It is then required to carry out resource manipulation (resource smoothing or resource constrained scheduling). In such cases, FARB can be utilized as a useful means of estimating the reliability of project completion time after resource manipulation. How such resource manipulation can be done is not within the scope of this paper and could be the subject of another study. It should also be noted that in this paper the calculated project completion time is based on normal (least cost) activities durations. A problem associated with reducing the project duration, referred to as the project compression problem, may arise as another sub-problem of total planning problem (Carmichael 2006). Project compression may be considered as a part of an iterative analysis attached on planning when the project completion time should meet a deadline. Shortening the durations of critical activities (activity compression) may lead to project compression which often comes at cost. There is, therefore, a trade-off between shortening the project completion time and increasing the project cost. In practice, such trade-off is not easy due to the availability of cost-duration data for any activity, and the accuracy of this data. In the circumstances where such data is available, FARB can be utilized as a useful means of estimating the reliability of project completion time after compression. How compression can be done is beyond the scope of this paper and could be the subject of another study.

A number of further developments from the present paper are possible; for example, time dependent problems. These problems move beyond the static methods discussed here to dynamic methods that change with times. In this sense the reliability of the project completion time should be developed as a function of time. Further extension could be made to develop methods for other engineering reliability problems. The verification of the method proposed in this paper was conducted on an example project obtained from the literature. Further assessment of the method could be carried out on different projects to enlarge the sample, and provide further evidence on the merits of method proposed in the paper.

Conclusions

A fast and accurate method for evaluating the reliability of project completion time for large construction projects has been developed using reliability theory. An exemplar project has been analysed to show the application of the proposed method and to demonstrate its capabilities in predicting the reliability of project completion time. It has been found that the results of the proposed method are accurate compared to the results produced by the Monte Carlo simulation method with sufficient samples. It has also been found that the reliability of project completion time predicted by the proposed method for upper and lower bounds are very close to each other. Furthermore, it has been demonstrated that the proposed method significantly reduces the number of network paths to be analysed and hence the computational efforts compared to the narrow reliability bounds method. It can be concluded that the proposed method can serve as a fast and accurate tool for project managers and project planners in project planning, project re-planning, and project control phases.

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