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DUAL RELATIONS OF LINEAR THIN GENERAL SHELL THEORY

R. Karkauskas

1. Some fundamentals

We use the "classical" shell theory based upon the Love and Kirchhoff approximate assumption, that straight lines normal to the middle surface before deformation remain straight, normal to the middle surface and unchanged in length after deformation.

The errors due to this approximation are negligible for "thin shells" whose wall thickness is small compared to the "radius of curvature" $R_{\min}$ ($h/R_{\min} \leq 1/20$). On the other hand, we know, that for thin shells the normal stress σ_z is small compared to other normal stresses. Thus the shell wall is in a condition of plane stress.

As coordinate lines we will use the "lines of curvature" of a continuous curved surface such as the undisplaced middle surface of the shell wall, together with normals to this surface. These lines of curvature are defined as lines along which the twist is zero. It is shown in the theory of continuous surfaces that there are always at least two such systems of lines, and that these systems are orthogonal to each other, that is tangents to two such lines at the point where they intersect will be at right angles to each other. Fig. 1 shows a point o with the orthogonal lines of curvature labelled α_1 and α_2, passing through o. We take the coordinates of point o as α_1, α_2 and of the points, a, b, adjacent to o in the directions of increase of α_1, α_2, as $\alpha_1+d\alpha_1$, α_2 and α_1, $\alpha_2+d\alpha_2$ respectively, as indicated (Fig. 1). We introduce variable scale factors A_1, A_2 defined so that $A_1 d\alpha_1$ and $A_2 d\alpha_2$ are the distances measured along the curves between o and a and between o and b; we assume A_1, A_2 to be continuous functions of α_1, α_2. These functions are called "coefficients of the first quadratic form" of continuous surfaces. Let the equations of middle surface be completely described by the expressions:

$$x = x(\alpha_1, \alpha_2), \quad y = y(\alpha_1, \alpha_2), \quad z = z(\alpha_1, \alpha_2).$$

Thus for defining A_1, A_2 we have:

$$\begin{aligned} A_1 &= \sqrt{(\partial x / \partial \alpha_1)^2 + (\partial y / \partial \alpha_1)^2 + (\partial z / \partial \alpha_1)^2}, \\ A_2 &= \sqrt{(\partial x / \partial \alpha_2)^2 + (\partial y / \partial \alpha_2)^2 + (\partial z / \partial \alpha_2)^2}. \end{aligned} \qquad (1)$$

We next erect rectangular coordinate axes x, y, z with origin at o, taking the x and y axes tangent respectively to the orthogonal α_1, α_2 lines at o, as shown in Fig. 1. They are fixed axes, because the α_1, α_2 lines are fixed in the undisplaced middle surface.

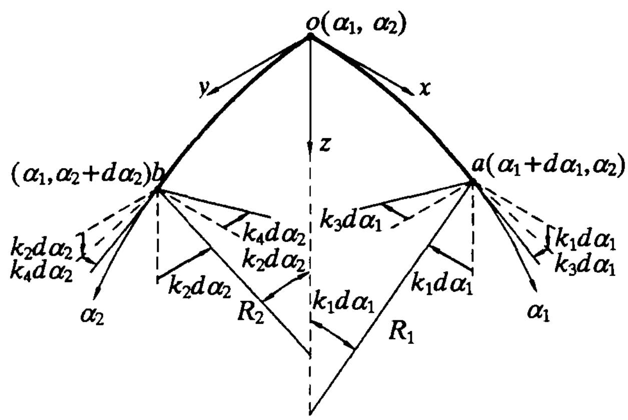
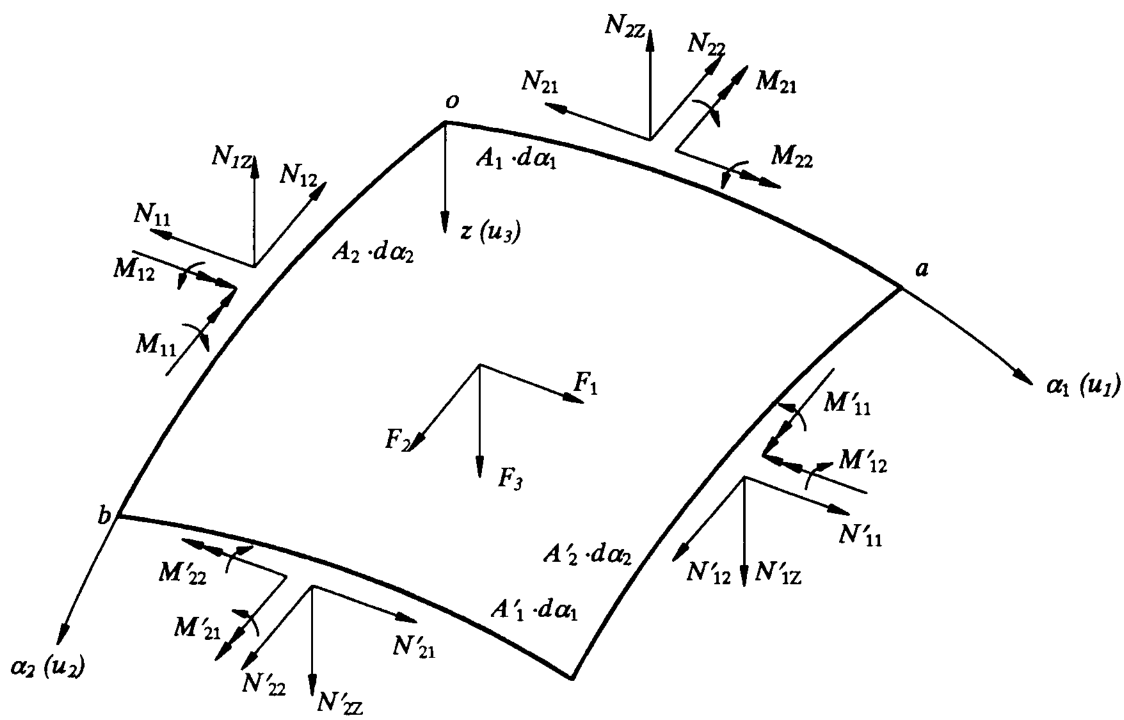


Fig. 1. Original positions

Similar triads of axes are erected at points a and b; they are indicated in Fig. 1 by the full straight lines through a and b. Due to the curvature of the surface and of the α_1, α_2 coordinate lines in the surface these triads will, in general, be rotated relative to the xyz directions, that is relative to the dotted lines shown in the figure.

These angles of rotation about the xyz directions are shown in Fig. 1. We take the rotations of the triad at a about the xyz directions to be 0, $k_1 d\alpha_1$, $k_3 d\alpha_1$ respectively, as shown. Similarly, the triad at b is



$$N'_{ij}=N_{ij}+\frac{\partial N_{ij}}{\partial \alpha_i}\cdot d\alpha_i;\quad M'_{ij}=M_{ij}+\frac{\partial M_{ij}}{\partial \alpha_i}\cdot d\alpha_i;\quad i=1,2;\quad j=1,2,z;$$

$$A'_1=A_1+\frac{\partial A_1}{\partial \alpha_2}\cdot d\alpha_2;\quad A'_2=A_2+\frac{\partial A_2}{\partial \alpha_1}\cdot d\alpha_1;$$

Fig. 2. Total forces and total moments on element sides

rotated through the angles $k_2 d\alpha_2$, 0, $k_4 d\alpha_2$ about the xyz directions, as shown. k_1, k_2, k_3 and k_4 are assumed to be continuous functions of α_1, α_2 having the physical meanings of a curvature of the surface in the α_1, α_2 directions and of the α_1, α_2 lines in the surface. Altogether it can be shown ([1]) that:

$$k_1 = A_1 / R_1, \qquad k_2 = A_2 / R_2,$$
$$k_3 = -\frac{1}{A_2}\frac{\partial A_1}{\partial \alpha_2}, \quad k_4 = -\frac{1}{A_1}\frac{\partial A_2}{\partial \alpha_1}. \qquad (2)$$

These relations will be found useful.

2. General shell equilibrium equations

The equilibrium equations are found by considering the equilibrium of an infinitesimal element of a continuous curved surface, of the dimensions $A_1 d\alpha_1$ and $A_2 d\alpha_2$, as illustrated in Fig. 2. For simplicity, we show only the undisplaced middle surface of the shell, with all the forces acting on the element and the moments about the middle surface.

We define the N's and M's shown as forces and moments per unit length of section at the middle surface. The forces and moments are designated by two subscripts, the first of them indicates the direction of the normal to the side on which the stresses act, and the second the direction of the axis.

The components of the external distributed load which act on the surface are shown in Fig. 2; $F_j(\alpha_1, \alpha_2)$, $j=1, 2, 3$ is a distributed force per unit area acting in the positive direction of the axis. To the approximation which we obtain by using the Love - Kirchhoff assumption, we can not distinguish between such a force acting on the upper or lower surface.

The information given in Figs. 1 and 2 can now be used to write down the six equations of equilibrium. We need equations of equilibrium of forces in the α_1, α_2, z directions $\sum F_{\alpha_1}$, $\sum F_{\alpha_2}$, $\sum F_z = 0$ and of moments of the forces about axes in the α_1, α_2, z directions, say, through the centre of the element, $\sum M_{\alpha_1}$, $\sum M_{\alpha_2}$, $\sum M_z = 0$.

From the above discussion, it is evident that total forces equilibrium in the α_1 directions requires that:

$$-N_{11}A_2d\alpha_2 + N_{11}^{'}A_2^{'}d\alpha_2 - N_{21}A_1d\alpha_1 + N_{21}^{'}A_1^{'}d\alpha_1$$
$$+N_{22}^{'}A_1^{'}d\alpha_1k_4d\alpha_2 - N_{12}^{'}A_2^{'}d\alpha_2k_3d\alpha_1$$
$$-N_{1z}^{'}A_2^{'}d\alpha_2k_1d\alpha_1 - F_1A_1d\alpha_1A_2d\alpha_2 = 0.$$

Using information given in Fig. 2 and Eqs. (2) and dividing by the factor $d\alpha_1d\alpha_2$ (terms containing more increments of course being ignored as small quantities of higher order), the first equation is presented:

$$\frac{\partial(A_2N_{11})}{\partial\alpha_1} + \frac{\partial(A_1N_{21})}{\partial\alpha_2} + k_4A_1N_{22} - k_3A_2N_{12}$$
$$-k_1A_2N_{1z} + F_1A_1A_2 = 0.$$

The second equation is the same as this with subscripts 1 and 2, 3 and 4 interchanged:

$$\frac{\partial(A_1N_{22})}{\partial\alpha_2} + \frac{\partial(A_2N_{12})}{\partial\alpha_1} + k_3A_2N_{11} - k_4A_1N_{21}$$
$$-k_2A_1N_{2z} + A_2A_1F_2 = 0.$$

The third equilibrium equation (the most important of them all, since it represents equilibrium of forces tending to deform the shell in its weakest direction, the direction of the small thickness) for the z direction is:

$$-N_{1z}A_2d\alpha_2 + N_{1z}^{'}A_2^{'}d\alpha_2 - N_{2z}A_1d\alpha_1 + N_{2z}^{'}A_1^{'}d\alpha_1$$
$$+N_{11}^{'}A_2^{'}d\alpha_2k_1d\alpha_1 + N_{22}^{'}A_1^{'}d\alpha_1k_2d\alpha_2$$
$$+F_3A_1d\alpha_1A_2d\alpha_2 = 0.$$

Cancelling $d\alpha_1d\alpha_2$ and using information given in Fig. 2, these can be written in the form:

$$k_1A_2N_{11} + k_2A_1N_{22} + \frac{\partial(A_2N_{1z})}{\partial\alpha_1} + \frac{\partial(A_1N_{2z})}{\partial\alpha_2}$$
$$+A_1A_2F_3 = 0.$$

The fourth equation of equilibrium, stating that the moments of forces about the α_2 direction up to zero, becomes:

$$\frac{\partial(A_2M_{11})}{\partial\alpha_1} + \frac{\partial(A_1M_{21})}{\partial\alpha_2} + k_4A_1M_{22} - k_3A_2M_{12}$$
$$-A_1A_2N_{1z} = 0.$$

The fifth equation is the same as this with subscripts 1 and 2, 3 and 4 interchanged:

$$\frac{\partial(A_1M_{22})}{\partial\alpha_2} + \frac{\partial(A_2M_{12})}{\partial\alpha_1} + k_3A_2M_{11} - k_4A_1M_{21}$$
$$-A_2A_1N_{2z} = 0.$$

The last equation of equilibrium, of moments about the z axis, becomes:

$$\frac{k_1}{A_1}M_{12} - \frac{k_2}{A_2}M_{21} + N_{21} - N_{12} = 0.$$

This equation is an identity. It will be seen that as we have the integral expressions of the initial forces:

$$\frac{k_1}{A_1}\int_{-0.5h}^{0.5h}\sigma_{12}\left(1-\frac{k_2}{A_2}z\right)zdz - \frac{k_2}{A_2}\int_{-0.5h}^{0.5h}\sigma_{21}\left(1-\frac{k_1}{A_1}z\right)zdz$$
$$+\int_{-0.5h}^{0.5h}\sigma_{21}\left(1-\frac{k_1}{A_1}z\right)dz - \int_{-0.5h}^{0.5h}\sigma_{12}\left(1-\frac{k_2}{A_2}z\right)dz$$
$$= \int_{-0.5h}^{0.5h}(\sigma_{21}-\sigma_{12})\left(1-\frac{k_1}{A_1}z\right)\left(1-\frac{k_2}{A_2}z\right)dz = 0,$$

because $\sigma_{21} = \sigma_{12}$.

We can now solve for N_{1z} and N_{2z} from the fourth and fifth equations, which represent equilibrium of moments about the α_2 and α_1 directions. With these modifications and replacing k_1, k_2, k_3, k_4 by Eqs.(2), the general shell equilibrium equations become:

$$-\frac{\partial(A_2N_{11})}{\partial\alpha_1} + \frac{\partial A_2}{\partial\alpha_1}N_{22} - \frac{\partial A_1}{\partial\alpha_2}N_{12} - \frac{\partial(A_1N_{21})}{\partial\alpha_2}$$
$$+\frac{1}{R_1}\frac{\partial(A_2M_{11})}{\partial\alpha_1} - \frac{1}{R_1}\frac{\partial A_2}{\partial\alpha_1}M_{22} + \frac{1}{R_1}\frac{\partial A_1}{\partial\alpha_2}M_{12}$$
$$+\frac{1}{R_1}\frac{\partial(A_1M_{21})}{\partial\alpha_2} = A_1A_2F_1;$$

$$-\frac{\partial(A_1N_{22})}{\partial\alpha_2} + \frac{\partial A_1}{\partial\alpha_2}N_{11} - \frac{\partial A_2}{\partial\alpha_1}N_{21} - \frac{\partial(A_2N_{12})}{\partial\alpha_1}$$
$$+\frac{1}{R_2}\frac{\partial(A_1M_{22})}{\partial\alpha_2} - \frac{1}{R_2}\frac{\partial A_1}{\partial\alpha_2}M_{11} + \frac{1}{R_2}\frac{\partial A_2}{\partial\alpha_1}M_{21}$$
$$+\frac{1}{R_2}\frac{\partial(A_2M_{12})}{\partial\alpha_1} = A_1A_2F_2; \quad (3)$$

Table 1. Differential operator of equilibrium equations

$-\frac{\partial(A_2\cdots)}{\partial\alpha_1}$	$\frac{\partial A_2}{\partial\alpha_1}\cdots$	$-\frac{\partial A_1}{\partial\alpha_2}\cdots$ $-\frac{\partial(A_1\cdots)}{\partial\alpha_2}$	$\frac{1}{R_1}\frac{\partial(A_2\cdots)}{\partial\alpha_1}$	$-\frac{1}{R_1}\frac{\partial A_2}{\partial\alpha_1}\cdots$	$\frac{1}{R_1}\frac{\partial A_1}{\partial\alpha_2}\cdots$ $+\frac{1}{R_1}\frac{\partial(A_1\cdots)}{\partial\alpha_2}$
$\frac{\partial A_1}{\partial\alpha_2}\cdots$	$-\frac{\partial(A_1\cdots)}{\partial\alpha_2}$	$-\frac{\partial A_2}{\partial\alpha_1}\cdots$ $-\frac{\partial(A_2\cdots)}{\partial\alpha_1}$	$-\frac{1}{R_2}\frac{\partial A_1}{\partial\alpha_2}\cdots$	$\frac{1}{R_2}\frac{\partial(A_1\cdots)}{\partial\alpha_2}$	$\frac{1}{R_2}\frac{\partial A_2}{\partial\alpha_1}\cdots$ $+\frac{1}{R_2}\frac{\partial(A_2\cdots)}{\partial\alpha_1}$
$-\frac{A_1A_2}{R_1}\cdots$	$-\frac{A_1A_2}{R_2}\cdots$	0	$-\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial(A_2\cdots)}{\partial\alpha_1}\right)$ $+\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial A_1}{\partial\alpha_2}\cdots\right)$	$-\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial(A_1\cdots)}{\partial\alpha_2}\right)$ $+\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial A_1}{\partial\alpha_1}\cdots\right)$	$-\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial(A_1\cdots)}{\partial\alpha_2}\right)$ $-\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial(A_2\cdots)}{\partial\alpha_1}\right)$ $-\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial A_1}{\partial\alpha_2}\cdots\right)$ $-\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial A_2}{\partial\alpha_1}\cdots\right)$

$$-\frac{A_1A_2}{R_1}N_{11}-\frac{A_2A_1}{R_2}N_{22}-\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial(A_2M_{11})}{\partial\alpha_1}\right)$$
$$-\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial(A_1M_{21})}{\partial\alpha_2}\right)+\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial A_2}{\partial\alpha_1}M_{22}\right)$$
$$-\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial A_1}{\partial\alpha_2}M_{12}\right)-\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial(A_1M_{22})}{\partial\alpha_2}\right)$$
$$-\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial(A_2M_{12})}{\partial\alpha_1}\right)+\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial A_1}{\partial\alpha_2}M_{11}\right)$$
$$-\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial A_2}{\partial\alpha_1}M_{21}\right)=A_1A_2F_3.$$

The differences between N_{12} and N_{21} or between M_{12} and M_{21} are minor quantities for thin shells and can be ignored for most purposes. In this case we would have $N_{12}= N_{21}$ and $M_{12}= M_{21}$. Then the strained state of the shell are defined by the six-dimensional vector-function of forces

$$\mathbf{S}\equiv(N_{11},\ N_{22},\ N_{12}=N_{21},\ M_{11},\ M_{22},\ M_{12}=M_{21})^{\mathrm{T}}.$$

These forces are of course in general functions of both α_1 and α_2.

The load is characterized by the three-dimensional vector-function of distributed load

$$\mathbf{F}\equiv(F_1,\ F_2,\ F_3)^{\mathrm{T}}.$$

If the vectors S and F are chosen as indicated above, then the Eqs. (3) of a general shell have the form

$$[\mathrm{A}]\mathbf{S}=\mathbf{F}. \qquad (4)$$

where [A] is the differential operator of the equilibrium equations. It is shown in Table 1.

3. General shell geometric equations

The geometric equations, which define the connection between displacements and deformations, can be obtained purely formally since the operators of the equilibrium equations and kinematic compatibility are adjoints. Thus we have

$$[\mathrm{A}]^{\mathrm{T}}\mathbf{u}=\mathbf{q}. \qquad (5)$$

Table 2. Differential operator of geometric equations

	$A_2\dfrac{\partial\ldots}{\partial\alpha_1}$	$\dfrac{\partial A_1}{\partial\alpha_2}\cdot\ldots$	$-\dfrac{A_1A_2}{R_1}\cdot\ldots$
	$\dfrac{\partial A_2}{\partial\alpha_1}\cdot\ldots$	$A_1\dfrac{\partial\ldots}{\partial\alpha_2}$	$-\dfrac{A_1A_2}{R_2}\cdot\ldots$
	$A_1\dfrac{\partial\ldots}{\partial\alpha_2}-\dfrac{\partial A_1}{\partial\alpha_2}\cdot\ldots$	$A_2\dfrac{\partial\ldots}{\partial\alpha_1}-\dfrac{\partial A_2}{\partial\alpha_1}\cdot\ldots$	0
$\dfrac{1}{A_1A_2}$	$-\dfrac{A_2}{R_1}\dfrac{\partial\ldots}{\partial\alpha_1}$	$-\dfrac{1}{R_2}\dfrac{\partial A_1}{\partial\alpha_2}\cdot\ldots$	$-A_2\dfrac{\partial}{\partial\alpha_1}\left(\dfrac{1}{A_1}\dfrac{\partial\ldots}{\partial\alpha_1}\right)-\dfrac{1}{A_2}\dfrac{\partial A_1}{\partial\alpha_2}\dfrac{\partial\ldots}{\partial\alpha_2}$
	$-\dfrac{1}{R_1}\dfrac{\partial A_2}{\partial\alpha_1}\cdot\ldots$	$-\dfrac{A_1}{R_2}\dfrac{\partial\ldots}{\partial\alpha_2}$	$-A_1\dfrac{\partial}{\partial\alpha_2}\left(\dfrac{1}{A_2}\dfrac{\partial\ldots}{\partial\alpha_2}\right)-\dfrac{1}{A_1}\dfrac{\partial A_2}{\partial\alpha_1}\dfrac{\partial\ldots}{\partial\alpha_1}$
	$-\dfrac{A_1}{R_1}\dfrac{\partial\ldots}{\partial\alpha_2}+\dfrac{1}{R_1}\dfrac{\partial A_1}{\partial\alpha_2}\cdot\ldots$	$-\dfrac{A_2}{R_2}\dfrac{\partial\ldots}{\partial\alpha_1}+\dfrac{1}{R_2}\dfrac{\partial A_2}{\partial\alpha_1}\cdot\ldots$	$-A_1\dfrac{\partial}{\partial\alpha_1}\left(\dfrac{1}{A_1}\dfrac{\partial\ldots}{\partial\alpha_2}\right)-A_2\dfrac{\partial}{\partial\alpha_2}\left(\dfrac{1}{A_2}\dfrac{\partial\ldots}{\partial\alpha_1}\right)+\dfrac{1}{A_1}\dfrac{\partial A_1}{\partial\alpha_2}\dfrac{\partial\ldots}{\partial\alpha_1}+\dfrac{1}{A_2}\dfrac{\partial A_2}{\partial\alpha_1}\dfrac{\partial\ldots}{\partial\alpha_2}$

Here the vector-function **u** is the dual variable for the load **F**, and denotes the displacements. We designate the components in the α_1, α_2, z directions of the displacements of points in the middle surface of the shell by u_1, u_2, u_3, as indicated in the parentheses after the axis designations in Fig. 2. These displacements u_1, u_2, u_3 are of course in general functions of both α_1 and α_2. Then the vector

$$\mathbf{u}\equiv\left(u_1,\,u_2,\,u_3\right)^{\mathrm{T}}.$$

The vector-function **q** is the dual of the vector of forces S, and denotes the strains. We designate the components of the strains by Δ_{11}, Δ_{22}, Δ_{12}, χ_{11}, χ_{22}, χ_{12}. The first three terms are the "membrane" strains; Δ_{11}, Δ_{22} are the changes in length of the middle surface of an infinitesimal element in the α_1, α_2 directions, Δ_{12} is the change in the angle between the sides of element, whose normals were originally in the α_1 and α_2 directions. The last three terms are the "flexural" strains; χ_{11}, χ_{22} are the curvatures of the middle surface in the α_1, α_2 directions, χ_{12} is the twist of the surface.

The operator $[\mathrm{A}]^{\mathrm{T}}$ is the transpose of the operator of the equation of equilibrium. Altogether it can be shown ([2]) that:

$$\text{if}\quad a_{ij} = f(\alpha_1, \alpha_2)\frac{\partial^{m+n}\left(f(\alpha_1, \alpha_2)u_i\right)}{\partial\alpha_1^m\partial\alpha_2^n},$$

$$\text{then}\quad a_{ji}^{\mathrm{T}} = (-1)^{m+n}\varphi(\alpha_1, \alpha_2)\frac{\partial^{m+n}\left(f(\alpha_1, \alpha_2)u_i\right)}{\partial\alpha_1^m\partial\alpha_2^n}. \qquad (6)$$

Using Eqs. (6), we find the operator $[A]^{\mathrm{T}}$, given in Table 2. This information can now be used to write down the six equations of kinematic compatibility:

$$\Delta_{11} = \frac{1}{A_1}\frac{\partial u_1}{\partial\alpha_1} + \frac{u_2}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2} - \frac{u_3}{R_1},$$

$$\Delta_{22} = \frac{u_1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1} + \frac{1}{A_2}\frac{\partial u_2}{\partial\alpha_2} - \frac{u_3}{R_2},$$

$$\Delta_{12} = \frac{1}{A_2}\frac{\partial u_1}{\partial\alpha_2} - \frac{u_1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2} + \frac{1}{A_1}\frac{\partial u_2}{\partial\alpha_1} - \frac{u_2}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}; \qquad (7a)$$

$$\chi_{11} = -\frac{1}{A_1R_1}\frac{\partial u_1}{\partial\alpha_1} - \frac{u_2}{A_1A_2R_2}\frac{\partial A_1}{\partial\alpha_2} + \frac{1}{A_1^3}\frac{\partial A_1}{\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_1} - \frac{1}{A_1^2}\frac{\partial^2 u_3}{\partial\alpha_1^2} - \frac{1}{A_1A_2^2}\frac{\partial A_1}{\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_2},$$

$$\chi_{22} = -\frac{u_1}{A_1A_2R_1}\frac{\partial A_2}{\partial\alpha_1} - \frac{1}{A_2R_2}\frac{\partial u_2}{\partial\alpha_2} + \frac{1}{A_2^3}\frac{\partial A_2}{\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_2} - \frac{1}{A_2^2}\frac{\partial^2 u_3}{\partial\alpha_2^2} - \frac{1}{A_1^2A_2}\frac{\partial A_2}{\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_1},$$

$$\chi_{12} = -\frac{1}{A_2R_1}\frac{\partial u_1}{\partial\alpha_2} + \frac{u_1}{A_1A_2R_1}\frac{\partial A_1}{\partial\alpha_2} - \frac{1}{A_1R_2}\frac{\partial u_2}{\partial\alpha_1} + \frac{u_2}{A_1A_2R_2}\frac{\partial A_2}{\partial\alpha_1} + \frac{1}{A_1^2A_2}\frac{\partial A_1}{\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_2} + \frac{1}{A_1A_2^2}\frac{\partial A_2}{\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_1} - \frac{2}{A_1A_2}\frac{\partial^2 u_3}{\partial\alpha_1\partial\alpha_2} + \frac{1}{A_1^2A_2}\frac{\partial A_1}{\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_1} + \frac{1}{A_1A_2^2}\frac{\partial A_2}{\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_2}. \qquad (7b)$$

These are the most general thin shell geometric equations.

The equations (4) and (5) can be converted to shells of specific geometric shapes by substituting in them the corresponding values of the geometrical functions.

4. Geometric functions

For clarity, we write out the most common way of defining the parametres α_1, α_2 and the resulting values of the functions (scale factors) A_1, A_2 for the types of shells of usual practical interest.

Fig. 3 shows the simplest case of a flat plate using rectangular coordinates $\alpha_1=x$, $\alpha_2=y$, for which the scale factors $A_1=1$ and $A_2=1$.

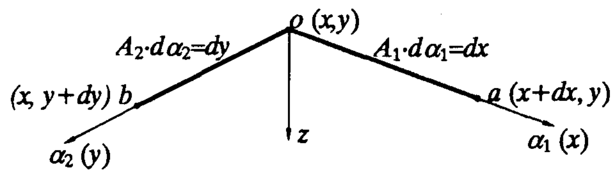


Fig. 3. Flat plate

In Fig. 4, using polar coordinates $\alpha_1=r$, $\alpha_2=\theta$, the same is true in the radial direction, $A_1=1$. In the angular direction, a small change $d\theta$ in the parameter $\alpha_2=\theta$ produces an arc length of $rd\theta = A_2\, d\alpha_2$, giving $A_2=r$.

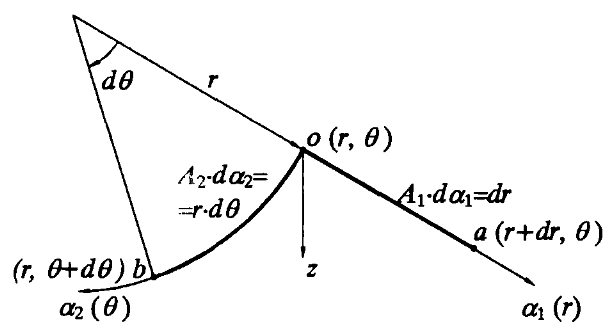


Fig. 4. Axi-rotational plate

Fig. 5 for the right cylindrical shell, using an angular coordinate, seems equally clear. In this case $\alpha_1=x$, $\alpha_2=\theta$, $A_1=1$ and $A_2=R_2$.

Fig. 6 shows the case of a conical shell, using distances along the axis $x=\alpha_1$, and rotations about the axis $\theta=\alpha_2$. In the x direction the actual distances along the middle surface are $x/\cos\gamma$, where γ is the cone angle; hence the scale factor $A_1=1/\cos\gamma$. In the $\alpha_2=\theta$ direction the length of the arc ab is $A_2\, d\alpha_2 = rd\theta = x\, tg\gamma\, d\theta$, and the scale factor $A_2 = x\, tg\gamma$.

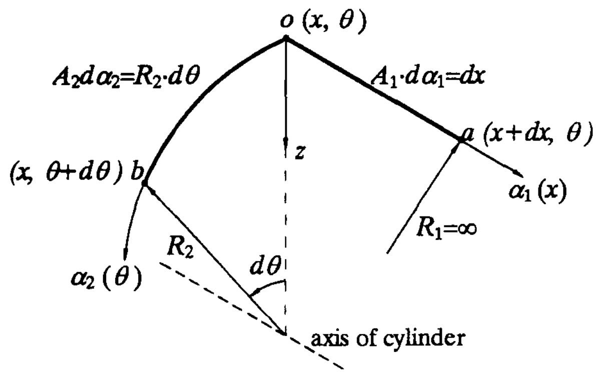


Fig. 5. Cylindrical shell

In the case of the axi-rotational shell, shown in Fig. 7, we take the coordinates as two angles, one angle of latitude $\alpha_1=\varphi$, and the second angle of longitude $\alpha_2=\theta$. Then the scale factors $A_1=R_1$ and $A_2=r=R_2 sin\varphi$. Here R_1, R_2 are the radii of curvatures in the α_1, α_2 directions.

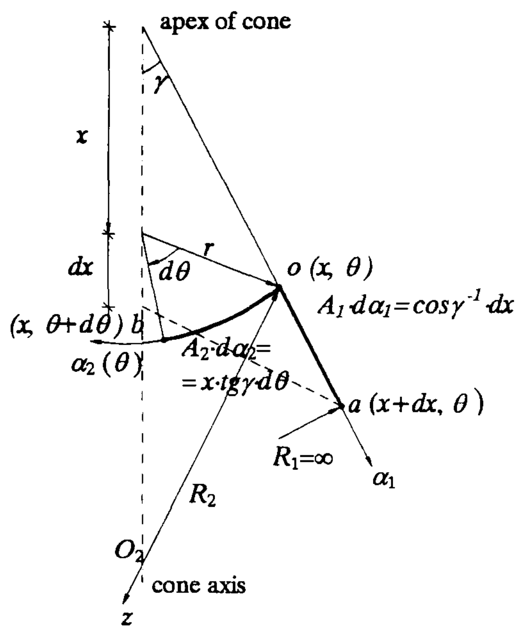


Fig. 6. Circular cone

Fig. 8 shows a double-curved shell, using rectangular coordinates, for which the scale factor $A_1=1$ and $A_2=1$.

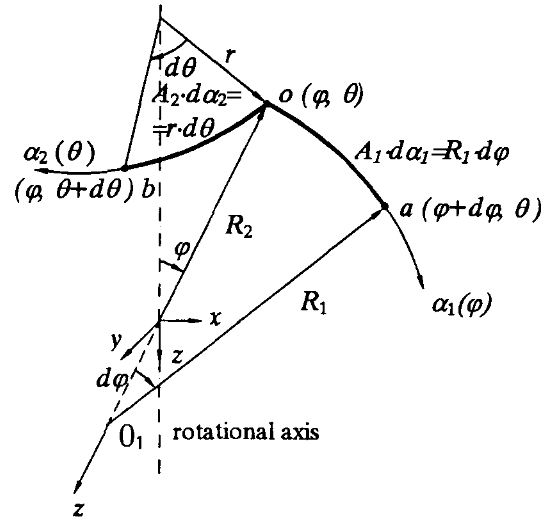


Fig. 7. Axi-rotational shell

In all these cases, the same resulting values of scale factors may be given, using the Eqs. (1). The equations of middle surface is then found as a continuous functions of α_1, α_2. For example, in the case of sphere (see Fig. 7 with $R_1=R_2=R$) equations of middle surface are:

$$x = R\sin\varphi\cos\Theta,\quad y = R\sin\varphi\sin\Theta,\quad z = -R\cos\varphi.$$

Then

$$A_1 = \sqrt{(R\cos\varphi\cos\Theta)^2 + (R\cos\varphi\sin\Theta)^2 + (R\sin\varphi)^2} = R,$$

$$A_2 = \sqrt{(R\sin\varphi\sin\Theta)^2 + (R\sin\varphi\cos\Theta)^2} = R\sin\varphi.$$

These values are defined in Fig. 7.

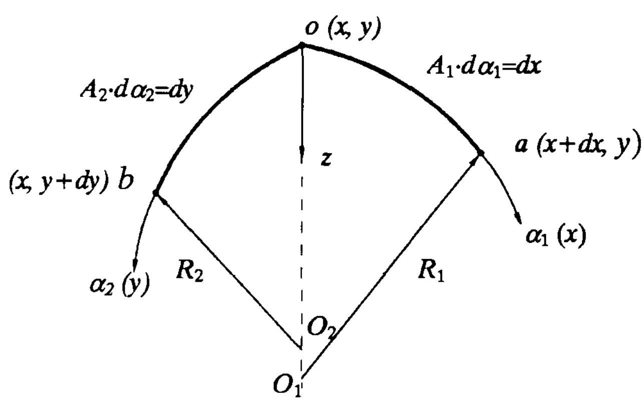


Fig. 8. Double-curved (general) shell

5. Conclusions

Equations (4) and (5) together with Eqs. (2) form the dual relations for general thin shell linear theory. They, and modifications of them, can be converted to theories for shells of specific geometric shapes merely by substituting in them the corresponding values of the geometric functions, such as those discussed previously.

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DUALIOS PRIKLAUSOMYBĖS TIESINĖJE PLONŲ APIBENDRINTOS FORMOS KEVALŲ TEORIJOJE

R. Karkauskas

S a n t r a u k a

Straipsnyje naudojant Kirchofo-Liavo hipotezę išvestos plono apibendrintos formos kevalo elemento dualios priklausomybės. Jas sudaro diferencialinės pusiausvyros ir geometrinės lygtys kreivalinijinėje koordinačių sistemoje. Pateiktos priklausomybių kvadratinės formos koeficientų reikšmės įvairiose koordinačių sistemose, kurios gali būti taikomos konkrečios formos kevalams skaičiuoti.

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