Mathematical Modelling and Analysis
Volume 13 Number 3, 2008, pages 413–420
Doi:10.3846/1392-6292.2008.13.413-420

APPLICATIONS OF 3D PCA METHOD FOR EXTRACTION OF MEAN SHAPE AND GEOMETRICAL FEATURES OF BIOLOGICAL OBJECTS SET

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Received October 23, 2007; revised May 15, 2008; published online September 9, 2008

Abstract. This article presents application of the Principal Component Analysis method for analysis of geometry of biological objects and computation of three dimensional anthropometric database. In this work as the biological objects the fifteen human femur bones were used. The geometry of each bone was obtained by using 3D structural light scanner. For PCA analysis all objects have to be described with the same FEM mesh. To achieve this, the modified fluid registration was used. PCA decomposes the set of 3D objects into mean geometry and individual features (empirical modes) describing deviations from mean value. In this paper the mean shape and features of real bones were presented and discussed.

Key words: 3D reconstruction, 3D scanner, anthropometric database, CAD, PCA (Principal Component Analysis), registration, reverse engineering.

1 Introduction

Nowadays, many engineering CAx techniques are applied not only in mechanics, but also in different disciplines like biomechanics, bioengineering, etc. This interdisciplinary research takes advantage of reverse engineering, three-dimensional modelling and simulation, and the Finite Element Method (FEM). The 3D virtual models have numerous applications such as visualization, medical diagnostics (virtual endoscopes), pre-surgical planning (simulations of surgical operations), FEM analysis, CNC machining, Rapid Prototyping, etc.

For using CAx tools, a 3D geometrical model is required. The main problem arises due to insufficient information in existing databases. That three dimensional knowledge about e.g. geometry of the bones does not exist. Traditional anthropometric database contains information only about some characteristic points (other parameters are not collected), when the set of the bones is described only in two dimensional space, by the collection of linear and angular dimensions (see Fig. 1). Usually data acquisition process is made with the usage of the conventional measurements equipment (e.g. calliper). If a new parameter between the given points (e.g.: angle, length, etc.) is needed, the completely new study and anthropometric measurement process must be done.

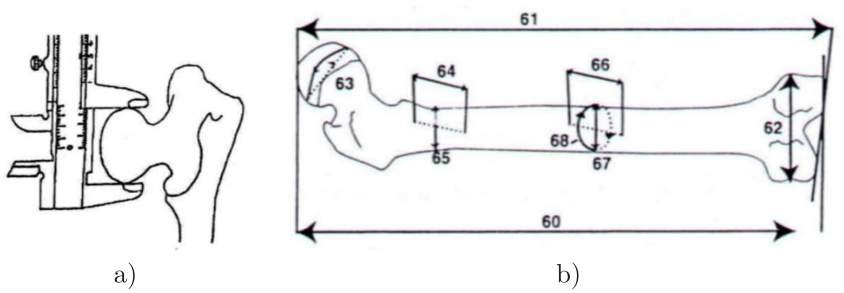


Figure 1. The example of measurement process and view on a few characteristic points for femur bone.

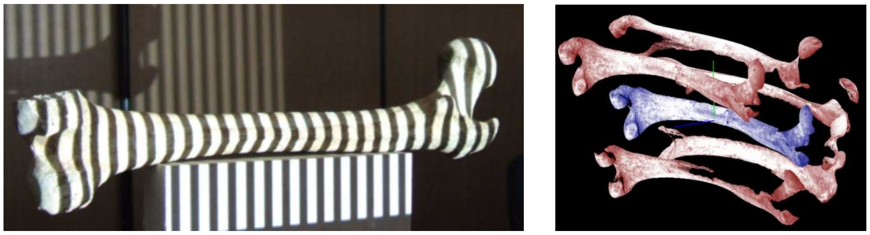
The solution of this problem is the application of the Reverse Engineering techniques for measurement process (to achieve 3D data) and modal analysis (PCA) to compute the 3D anthropometric database. These aspects are discussed further in details.

2 Data Acquisition by 3D Scanning

For construction of database the set of real bones was used. In this work 15 femur bones, including 6 female and 9 male, were used (see Fig. 2). They were obtained from Poznan University of Medical Science.

Figure 2. View of two female femur bones.

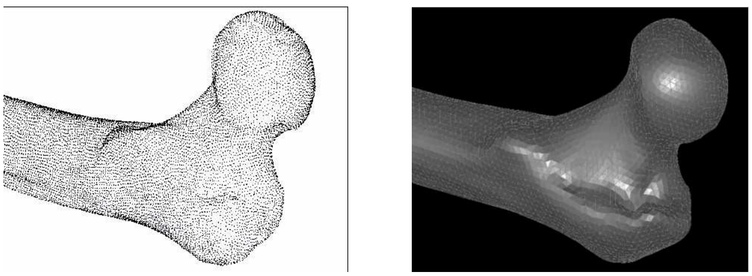
For acquisition of data the 3D structural light scanner was used. To obtain full 3D geometry, each bone was measured from 15 different directions with accuracy of 0.05 mm. The result of measurement is a set of 3D point clouds (see Fig. 3), which must be registered, describing the orientation and connection between clouds.



a) b)

Figure 3. Data acquisition process on 3D structural light scanner and a set of point clouds.

The effect of registration is one point cloud which contains about 1.5 million points. As the final step a 3D triangle surface grid is generated. These results are presented in Fig. 4.



a) b)

Figure 4. Complete point cloud with 1.5 million points and the final triangle surface grid.

3 Three-Dimensional Modal Shape Description (Principal Component Analysis)

The next problem after the acquisition of the collection of 3D geometries of bones (CAD models) is to find the answer on the question how to work with that large amount of data. A possible solution of this problem is the application of methods for reduction of data size without losing the high quality of geometry. Various modal analysis methods are used, the examples are given by mathematical (spherical harmonics [8], Fourier transformation), physical (vibration modes [7]) and empirical Principal Component Analysis (PCA) [3] methods.

In this paper, for creation of 3D anthropometric database, we use the low-dimensional reconstruction based on the Principal Component Analysis (also known as Karhunen-Loeve Decomposition, POD) [2]. PCA provides a relevant set of basis functions, which allow identification of a low-dimensional subspace. Using the KL expansion, a given statistical process is described with the minimum number of uncorrelated modes (principal components). The resulting coordinate system (defined by the eigenfunctions of the correlation matrix) is optimal in the sense that the mean-square error resulting from a finite representation of the process is minimized.

In the example presented below, the program used in CFD (*Computational Fluid Dynamics*) modal analysis is adapted [5]. The shape of every object is represented in the database as the 3D FEM grid. Each FEM grid is described by a vector (3.1):

$$S_i = [s_{i1}, s_{i2}, \ldots, s_{iN}]^T, \quad i = 1, 2, \ldots, M, \tag{3.1}$$

where $s_{ij} = (x, y, z)$ describes coordinates of the nodes (FEM grid) in the Cartesian coordinates system, M is the number of the objects that are in database, N is the number of the FEM nodes of a single object.

After that the mean shape $\bar{S}$ and covariance matrix C are computed (3.2):

$$\bar{S} = \frac{1}{M}\sum_{i=1}^{M} S_i, \quad C = \frac{1}{M}\sum_{i=1}^{M} \tilde{S}_i \tilde{S}_i^{\,T}. \tag{3.2}$$

The differences between the obtained mean shape and the object are described by the deformation vector $\tilde{S}_i = S_i - \bar{S}$. The statistical analysis of the deformation vectors gives us the information about the empirical modes. Modes represent various features of the object, for example geometrical (shape), physical (density), displacement and rotation.

Only a few first modes carry the most important information, therefore each original object S_i is reconstructed by using some K principal components (3.3):

$$S_i = \bar{S} + \sum_{k=1}^{K} a_{ki}\Psi_k, \quad i = 1, 2, \ldots, M, \tag{3.3}$$

where Ψ_k is an eigenvector representing the orthogonal mode (the feature computed from database), a_{ik} is coefficient of that eigenvector.

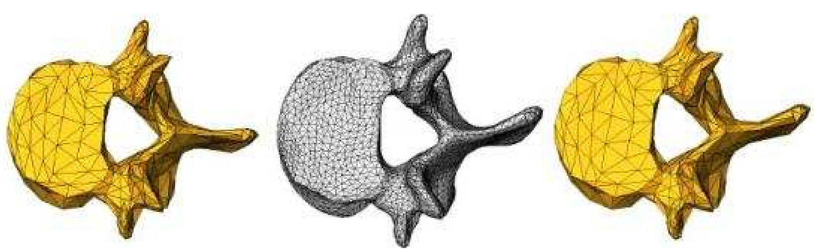


Figure 5. Triangle grid deformation: base vertebra (left side), input vertebra (middle), base grid on geometry of the new objects (right side).

The Principal Component Analysis requires the same topology of the FEM mesh (the same number of nodes, matrix connection, etc.) for each of the objects. To achieve this, every new object added to database, must be registrated. In this work the fluid registration was used [6]. The goal of registration is to put the base grid onto geometry of new objects (see Fig. 5).

For this registration the modified Navier-Stokes equation in penalty function formulation is used (3.4):

$$\underbrace{\dot{V}_i + V_{i,j}V_j - \frac{1}{Re}V_{i,jj} + \frac{\varepsilon - \lambda}{\rho}V_{j,ji} +}_{\text{existing numerical code [4]}} \underbrace{(f-g)f_{,i}}_{\text{source segment [1]}} = 0, \qquad (3.4)$$

where ρ is the fluid density, V_i is the velocity component, Re is the Reynolds number, λ is the bulk viscosity. In this application parameters ε and λ are used to control the fluid compressibility. The displacements of the nodes are computed from integration of the velocity field. The parameters of the flow are the same as for the compressible fluid with varying viscosity.

The computed flow field provides information about translations of the nodes. We obtain the dislocation of nodes of the base grid on the new geometry. This procedure is repeated for all bones.

4 PCA Analysis of Real Database

The database used in analysis contains 15 femur bones. Each bone has different geometry and is described by triangle surface grid (Fig. 6) with the same structure (14 thousand nodes, 30 thousand elements).

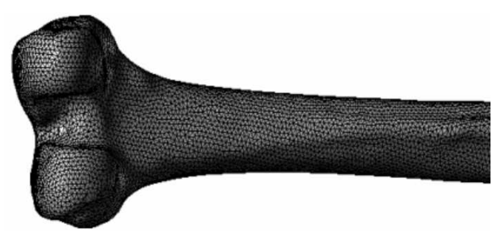


Figure 6. Triangle surface grid of femur bone.

Table 1. Participation of the modes in decomposition.

Mode no.	Participation of the mode [%]	Total participation of the modes	Mode no.	Participation of the mode [%]	Total participation of modes
1	74.921241	74.9212416	9	0.5866122	98.4379894
2	10.543835	85.4650767	10	0.4796167	98.9176061
3	4.2699519	89.7350286	11	0.3301463	99.2477523
4	3.3128685	93.0478971	12	0.3080968	99.5558492
5	1.6659793	94.7138765	13	0.2516839	99.8075330
6	1.4234329	96.1373093	14	0.1924670	100.000000
7	1.0359034	97.1732127	15	0.0000000	100.000000
8	0.6781645	97.8513772			

For this database the Principal Component Analysis was done. The result of this operation is the mean object, fifteen modes and the coefficients. The first fourteen modes include 100% of information about the decomposed geometry (see Tab. 1). The fifteenth mode contains only a numerical noise and it is not used for further calculations.

Modes describe the features of the femur bones (see, Fig. 7). The first mode describes the change of the length of the femur bone. The second mode represents changing position of the head of the bone, the third mode describes a change of the arc of the shaft (body). The other modes describe more complex deformations. For example, the fourth mode describes a change of the position of the greater trochanter and lesser trochanter and also the thickness of the shaft (body).

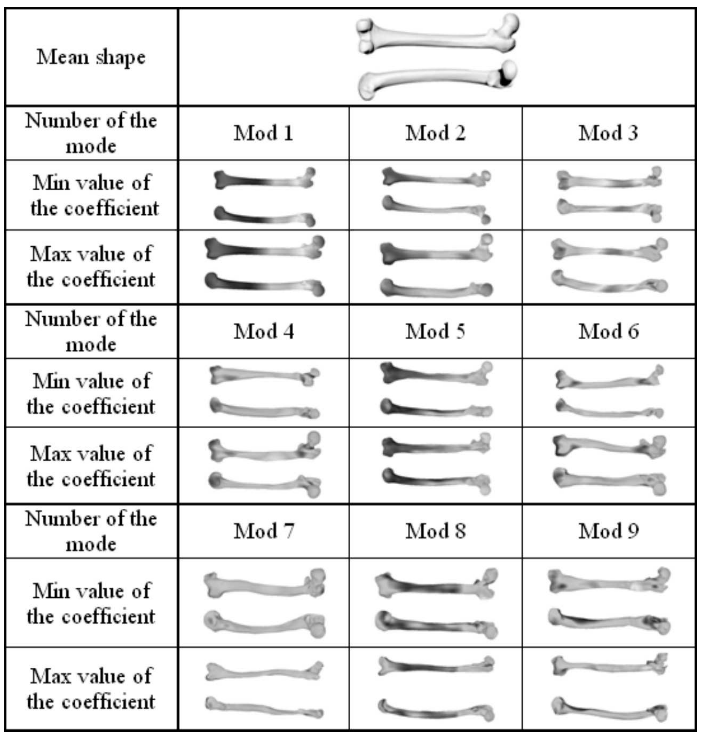


Figure 7. 3D visualization of mean value and first nine modes of femur bones (anterior and posterior view)

Study of the values of coefficients gives us an additional information about the analyzed bones. For presented database we observe a correlation between the coefficient value of the first mode and gender. If coefficient value is with sign "-" it means that this bone is probably female (with one exception of bone no. 8), and if the coefficient value is positive, it means that this bone is male (see, Fig. 8).

Another interesting aspect of coefficients is the existence of an individual set of coefficient values for each of the bones, this aspect is analogous to fingers prints. For the reason that each bone has another geometry also they must have personal set of coefficient values. On Fig. 9 graphs of coefficient values

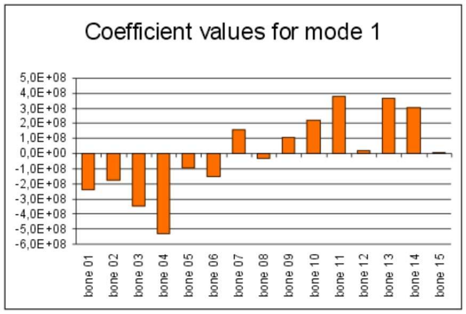


Figure 8. Correlation between coefficient value of first mode and gender: value "-" means female bone, value "+" means male bone.

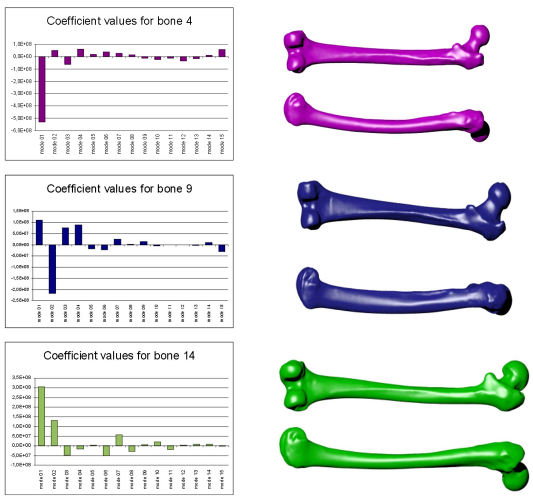


Figure 9. Correlation between coefficient value and geometry of the bone for three different femur bones (all pictures of bones are made in the same scale).

and three dimensional visualizations of the geometries for three different bones are presented.

5 Conclusions

Usage of virtual 3D models, CAx tools and knowledge from fluid mechanics gives us the new potential and new quality in others disciplines (e.g. medicine).

Application of three dimensional Principal Component Analysis makes possible the extraction of mean shape and geometrical features of biological object set. This method can be used in creation of full three dimensional anthropometric database. As the result of PCA analysis the 3D mean shape and the set of geometrical features (that describe principal deformations in analyzed group of objects) are obtained.

Math. Model. Anal., 13(3):413–420, 2008.

The advantage of three dimensional anthropometric database is the possibility of measurement of any dimension on the surface of the bone (3D surface of the mean shape) without new research and measurement processes on a set of bones.

Application of techniques from Reverse Engineering 3D scanners in data acquisition process, increase the numbers of collected data with high accuracy.

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