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SIGNIFICANCE OF AVERAGING COEFFICIENTS IN STABILITY ANALYSIS OF SHALLOW WAKE FLOWS [1]

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Abstract. Flows behind obstacles (such as islands) are shallow if the transverse scale of the flow is much larger than water depth. Field, laboratory and numerical data show that the flow pattern in shallow wakes exhibits a complex eddy-like motion. Experimental and theoretical analyses provide evidence for the presence of two-dimensional coherent structures in shallow water flows and show that the development of shallow wakes is different from the wakes in deep water due to the following reasons: first, the development of three-dimensional instabilities is prevented by limited water depth and second, bottom friction acts as a stabilizing mechanism for suppressing the transverse growth of perturbations. Several authors have used the linear and weakly nonlinear stability theory in order to understand when shallow flows become unstable. Two-dimensional depth-averaged Saint-Venant equations are usually used for the analysis. One of the main assumptions in shallow water theory is the independence of the velocity distribution on the vertical coordinate. In many cases, however, this assumption may not be valid. This paper presents an attempt to evaluate the influence of the assumption on the results of linear stability analysis of shallow wake flows with bottom friction. Momentum correction coefficients β_1 and β_2 are used in order to take into account the non-uniformity of the velocity distribution in the vertical direction. Linear stability calculations show that the stability boundary is quite sensitive to the variation of the parameters β_1 and β_2. The role of the linear and weakly nonlinear stability analysis on the formation of two-dimensional coherent structures in shallow water flows is discussed.

Key words: momentum correction coefficients, shallow wake flows, stability analysis

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1. Introduction

Shallow wake flows are flows behind obstacles (such as islands) with the transverse scale of the flow being much larger than the vertical scale (water depth). Experiments show that development of wakes in shallow water significantly differs from the development of wakes in deep water. This is linked to the fact that limited water depth has a strong influence on the development of flow instabilities. Bottom friction acts as a suppression factor for the growth of transverse perturbations. Moreover, evolution of three-dimensional instabilities is prevented due to small vertical scale. As it has been shown experimentally, the flow pattern of a shallow wake flow exhibits a complex eddy-like motion. Vortex structures observed in shallow water in many cases may resemble flow patterns in deep water, but in shallow water case the corresponding flow patterns can be observed at much larger values of the Reynolds number. For example, photograph No. 173 by Van Dyke [3] shows formation of eddies organized into a vortex street behind an obstacle in shallow water although the Reynolds number for this case is 10^7. Note that vortex street pattern in unbounded flows is limited to significantly smaller Reynolds numbers. Experimental and numerical data provide evidence for the presence of two-dimensional coherent structures in shallow water flows which are defined in [9] as connected large-scale fluid masses with a phase-correlated vorticity that extend uniformly over the fluid layer (with the exception of a boundary layer). Coherent structures observed in shallow flows are believed to be the end product of large scale flow instability [7]. The onset and initial development of two-dimensional coherent structures can be analyzed by means of linear and weakly nonlinear stability theory. Shallow wake flows, in particular flows behind islands in rivers and bays, are an object of growing interest from environmental point of view. Complex flows created by eddies can trap pollutants. Poor water circulation in a wake may lead to deposition of sediments. These two factors can result in poor water quality on the sheltered side of an island. Increased concentration of sediments and contaminants might affect marine culture causing, for example, fish disease and mortality. It is believed that the trapping of low-salinity Pearl river water in the sheltered areas led to intense stratification and resulted in deaths of marine inhabitants in Hong Kong in 1994 [5]. Keeping all above-said in mind it is clear that it is essential to know factors influencing flow patterns and, as a result, water circulation in shallow wakes. Due to this and, of course, in view of their environmental significance, shallow wake flows have been analyzed in the literature both theoretically and experimentally. Chen&Jirka [1] performed experimental studies of flows behind circular cylinders and showed that there are three different types of flow patterns observed in shallow wakes: vortex shedding, unsteady bubble and steady bubble. They also found that the type of flow pattern observed in a wake depends on a shallow wake stability parameter $S = c_f D/H$, where c_f is the bottom friction coefficient, D is the dimension of the obstacle (e.g. cylinder) and H is water depth. The parameter S was introduced earlier by Ingram&Chu [8]. The stability of shallow flows has been later studied by many authors [2, 4, 5, 15]. The independence of the flow characteristics on the vertical coordinate is an assumption that is usually made in the stability analysis of shallow flows. The assumption is linked to the fact that shallow water equations are depth-averaged equations. There are many cases, however, when the assumption may not be valid. Changes in flow geometry, flow regimes or roughness of the bottom boundary can lead to large deviations from the above-mentioned assumption [18, 19]. In order to take the non-uniformity of the velocity distribution into account, several authors have applied momentum correction coefficients [18, 19]. In particular, momentum correction coefficients are used in [6] for linear stability analysis of shallow mixing layers. Preliminary results on the influence of the non-uniformity of the velocity distribution on the stability of shallow wake flows under the rigid-lid assumption are presented in [12]. In this case water depth is constant so that free surface is assumed to be undisturbed. In the present paper the influence of averaging coefficients on the stability of shallow water flows is studied in detail. In addition, a weakly nonlinear model based on the complex Ginzburg-Landau equation is analyzed in order to describe the evolution of the most unstable mode above the threshold. Momentum correction coefficients β_1 and β_2 are used in this paper to calculate the stability boundary of the flow. The stability domains are calculated for the classical hyperbolic secant profile [2], given by

$$U(y) = 1 + \frac{2R}{1-R}\,\frac{1}{\cosh^2(\alpha y)}.$$

Here $R = (U_c - U_a)/(U_c + U_a)$ is the velocity ratio, $\alpha = \sinh^{-1}(1)$, U_c is the velocity on the centerline, U_a is the ambient velocity. The value of the parameter α is chosen so that the length scale of the flow is the wake half-width [2]. The role of the amplitude evolution equations (such as the Ginzburg-Landau model) on the formation of two-dimensional coherent structures in shallow water flows is discussed.

2. Problem Formulation

The governing equations are derived from the Euler equations by integrating them with respect to the vertical coordinate. The use of the inviscid Euler equations rather than the Navier-Stokes equations is justified as the value of Reynolds number Re is higher than 1000 for real island wakes. According to [2, 4] the stability analysis results are insensitive to the variation of Re when $Re >$ 1000. The rigid-lid assumption has been used in order to reduce the shallow water equations to a single equation with stream function of the flow acting as the unknown function. Under the rigid-lid assumption the gravity-driven free-surface flow is replaced by an equivalent pressure-driven flow between two parallel horizontal plates. The top plate is considered to have the friction coefficient equal to zero, while the bottom plate is considered to have friction coefficient c_f as the original channel. Ghidaoui&Kolyshkin [4] showed that the rigid-lid assumption can be applied for the case where the ratio of inertial force to gravity force is small. Experimental data [1] show that this assumption is usually satisfied for shallow wake flows. The governing equations for shallow flow under the rigid-lid assumption are [19]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + (2\beta_1 - 1)u\frac{\partial u}{\partial x} + (\beta_2 - 1)u\frac{\partial v}{\partial y} + \beta_2 v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} - \frac{c_f}{2h}u\sqrt{u^2+v^2}, \tag{2.2}$$

$$\frac{\partial v}{\partial t} + (\beta_2 - 1)v\frac{\partial u}{\partial x} + \beta_2 u\frac{\partial v}{\partial x} + (2\beta_3 - 1)v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} - \frac{c_f}{2h}v\sqrt{u^2+v^2}, \tag{2.3}$$

where x and y are the spatial coordinates, t is the time, u and v are the depth-averaged velocity components in the x and y directions respectively, h is water depth, c_f is the bottom friction coefficient defined by the equation [5]

$$\frac{1}{\sqrt{c_f}} = -4\log\Big(\frac{1.25}{4Re\sqrt{c_f}}\Big).$$

Shear stress at the boundary is modeled by the Chezy formula (see [16])

$$\tau_{wx} = \frac{1}{2}c_f\rho u\sqrt{u^2+v^2}, \quad \tau_{wy} = \frac{1}{2}c_f\rho v\sqrt{u^2+v^2},$$

where ρ is the density, τ_{wx} and τ_{wy} are the wall shear stresses along the x and y directions respectively. The coefficients β_1, β_2, and β_3 in equations (2.1)–(2.3) are the momentum correction coefficients which have been introduced in order to take into account non-uniformity of velocity distribution in the vertical direction. The meaning of the momentum correction coefficients can be explained by the following example of one-dimensional flow. Consider a fully developed (laminar or turbulent) flow in a long cylindrical pipe. In this case the velocity vector has only one nonzero component, v, in the longitudinal direction. The momentum can be calculated as follows

$$M_1 = \int_A \rho v^2\, dA,$$

where A is the cross-sectional area of the pipe. Hydraulic engineers often use simplified models [16] where the velocity of the fluid, V, is assumed to be constant over the cross section of the pipe. The momentum of the flow with uniform velocity V is given by

$$M_2 = \rho V^2 A.$$

Obviously, $M_1 \neq M_2$. Thus, when the velocity varies over the cross section of the pipe, a momentum correction factor β should be used before the average velocity V is introduced:

$$\int_A \rho v^2 dA = \beta \rho V^2 A.$$

Hence,

$$\beta = \frac{1}{A} \int_A \left(\frac{v}{V}\right)^2 dA.$$

Note that the correction coefficient β in this example is constant. However, it can be calculated only when the velocity distribution is known. Calculations presented in [16] show that $\beta = 2$ for laminar Poiseuille flow and $\beta = 1.02$ for turbulent flow of the form

$$\frac{v}{V_{max}} = \left(\frac{y_*}{a}\right)^{1/n}, \tag{2.4}$$

where $n = 7$, y_* is the distance from the wall and a is the radius of the pipe.

Similar idea is used for depth-averaged shallow water equations. The momentum correction coefficients are defined as follows:

$$\beta_1 = \frac{1}{hu^2} \int_{z_1}^{z_2} \tilde{u}^2 dz, \quad \beta_2 = \frac{1}{huv} \int_{z_1}^{z_2} \tilde{u}\tilde{v} dz, \quad \beta_3 = \frac{1}{hv^2} \int_{z_1}^{z_2} \tilde{v}^2 dz, \tag{2.5}$$

where $\tilde{u}$ and $\tilde{v}$ are the velocity components in the x and y directions respectively. It follows from (2.5) that the coefficients β_1, β_2 and β_3 are functions of the spatial coordinates x and y. In addition, the coefficients β_1, β_2 and β_3 can be calculated only when $\tilde{u}$ and $\tilde{v}$ are known. Experimental data (see [16]) show that for shallow wake flows the velocity distribution in the vertical direction varies only in a small boundary layer near the solid boundary. As a result, the coefficients β_1, β_2 and β_3 are expected to be small (slightly above 1). In the present paper we study the relative importance of the averaging coefficients on the stability characteristics by assuming that all the coefficients are fixed at some constant value. Introducing the stream function $\psi(x, y, t)$ defined by the relations

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

and eliminating the pressure p we rewrite equations (2.1)–(2.3) in the following form:

$$\begin{aligned}(\Delta\psi)_t &+ (2\beta_1 - \beta_2)(\psi_y \psi_{xy})_y - \beta_2(\psi_x \psi_{yy})_y + (\beta_2 - 1)(\psi_x \psi_{xy})_x \\ &+ \beta_2(\psi_{xx}\psi_y)_x - (2\beta_3 - 1)(\psi_x \psi_{xx})_y + \frac{c_f}{2h}\Delta\psi\sqrt{\psi_x^2 + \psi_y^2} \\ &+ \frac{c_f}{2h\sqrt{\psi_x^2 + \psi_y^2}}(\psi_y^2 \psi_{yy} + 2\psi_x \psi_y \psi_{xy} + \psi_x^2 \psi_{xx}) = 0,\end{aligned} \tag{2.6}$$

where Δ is the Laplacian in two dimensions and the subscripts indicate the derivatives with respect to the variables x and y. Suppose that the base flow

$$\boldsymbol{U} = \big(U(y), 0\big) \tag{2.7}$$

is perturbed and the perturbed solution to the equation (2.6) is assumed to be of the form

$$\psi = \psi_0 + \epsilon\psi_1 + \dots \tag{2.8}$$

where ϵ is a small parameter and $\psi_{0y} = U$. The smallness of the parameter ϵ (from a linear stability point of view) simply reflects the fact that the amplitude of the perturbation is assumed to be small in comparison with the amplitude of the base flow. Substituting (2.7) and (2.8) into (2.6) and linearizing the resulting equation in the neighborhood of the base flow (2.7) we obtain

$$\psi_{1xxt} + \psi_{1yyt} + (2\beta_1 - \beta_2)(U_y\psi_{1xy} + U\psi_{1xyy}) - \beta_2(U_y\psi_{1xy} + U_{yy}\psi_{1x}) \\ + \beta_2 U\psi_{1xxx} + \frac{c_f}{2h}(U\psi_{1xx} + 2U_y\psi_{1y} + 2U\psi_{1yy}) = 0. \tag{2.9}$$

According to the method of normal modes (see [14]) we seek the perturbed component ψ_1 of the stream function in the form

$$\psi_1(x, y, t) = \phi_1(y)e^{ik(x-ct)} + c.c. \tag{2.10}$$

where k is a wavenumber and $c = c_r + ic_i$ is a complex eigenvalue, "c.c." means "complex conjugate". Substituting (2.10) into (2.9) we obtain the linearized stability equation and boundary conditions in the form:

$$\begin{cases} \phi_1''\big((2\beta_1 - \beta_2)U - c + \frac{c_f}{ikh}U\big) + U_y(2\beta_1 - 2\beta_2\frac{c_f}{ikh})\phi_1' \\ \qquad + \big(k^2c - \beta_2 U_{yy} - k^2\beta_2 U - \frac{c_f}{2ih}kU\big)\phi_1 = 0, \\ \phi_1(\pm\infty) = 0. \end{cases} \tag{2.11}$$

We assume that the boundary conditions are specified at infinity since outer boundaries of real shallow flows are quite far from the obstacle and, therefore, it is natural to solve the problem in an unbounded domain.

3. Solution Method

It is known that for unbounded flows the spectrum consists of both a discrete and a continuous parts [14]. As discussed in [14], for practical and computational purposes it is often possible to use simpler formulation where a discretized approximation of the continuous spectrum is used. In addition, the analysis in [17] for the case of deep water wakes shows that the continuous spectrum cannot give a rise to unstable single modes. Taking into account the results of [14] and [17], only a discrete spectrum is analyzed in the present paper. The linear stability problem (2.11) is solved by a spectral collocation method based on the Chebyshev polynomials. Using the substitution

$$x = \frac{2}{\pi}\arctan(y), \quad y \in (-\infty, +\infty), \ \ x \in [-1; 1]$$

the interval (-∞; +∞) is mapped into the interval $(-1; 1)$. The solution $\phi(x)$ of the modified Rayleigh equation is sought in a form of the Chebyshev polynomial series:

$$\phi_1(x) = \sum_{n=0}^{N-1} a_n (1-x^2) T_n(x), \tag{3.1}$$

where a_n are unknown constants, and $T_n(x)$ is the n-order Chebyshev polynomial that has the form $T_n(x) = \cos(n \arccos(x))$. Since (3.1) contains a factor $(1-x^2)$, the boundary conditions in (2.11) are satisfied automatically for $x = \pm 1$. Using the collocation method and choosing the points

$$x_j = \cos\left(\frac{\pi j}{N+1}\right)$$

as the collocation points we obtain the generalized eigenvalue problem of the form

$$(A - \lambda B)a = 0, \tag{3.2}$$

where A and B are two complex-valued matrices and a is a vector of the form $a = (a_0, a_1, \ldots, a_{N-1})^T$. The elements of the matrices A and B have the form

$$a_{jn} = a_{jn}^{(r)} + i a_{jn}^{(i)}, \quad b_{jn} = b_{jn}^{(r)} + i b_{jn}^{(i)},$$

$$a_{jn}^{(r)} = \frac{c_f}{h}\Big(U q_{jn}^{(2)} + U_y q_{jn}^{(1)} - \frac{k^2}{2} U (1 - x_j^2) x_{jn}\Big),$$

$$a_{jn}^{(i)} = kU(2\beta_1 - \beta_2) q_{jn}^{(2)} + 2k(\beta_1 - \beta_2) U_y q_{jn}^{(1)} - k\beta_2 (k^2 U + U_{yy})(1 - x_j^2) x_{jn},$$

$$b_{jn}^{(r)} = -q_{jn}^{(2)} + k^2 (1 - x_j^2) x_{jn}, \quad b_{jn}^{(i)} = 0,$$

where

$$q_{jn}^{(1)} = \frac{2}{\pi}\big[-2x_j x_{jn} + n\sqrt{1-x_j^2}\sqrt{1-x_{jn}^2}\big] \cos^2\left(\frac{\pi}{2} x_j\right)$$

$$q_{jn}^{(2)} = -\frac{4}{\pi^2} \cos^4\left(\frac{\pi}{2} x_j\right)\Big[(2+n^2) x_{jn} - \big(3n x_j \sqrt{1-x_{jn}^2}\big)/\sqrt{1-x_j^2}\Big] - \frac{4}{\pi} \sin\left(\frac{\pi}{2} x_j\right) \cos^3\left(\frac{\pi}{2} x_j\right)\Big[-2x_j x_{jn} + n\sqrt{1-x_j^2}\sqrt{1-x_{jn}^2}\,\Big],$$

$$x_{jn} = \cos\frac{\pi j n}{N+1}, \quad j = 1, 2, \ldots N, \quad n = 1, 2, \ldots N.$$

Solving the generalized eigenvalue problem (3.2), for given c_f and k we obtain a set of eigenvalues c_m. The imaginary parts c_{im} of eigenvalues $c_i = c_{rm} + i c_{im}$ determine linear stability of the base flow. The flow is said to be linearly stable if the imaginary parts of all c_m are negative. If the imaginary part of the eigenvalue c_m of at least one mode is positive then a perturbation grows exponentially with time and the flow is said to be linearly unstable. Calculations show that for sufficiently large values of the friction coefficient c_f all eigenvalues have negative imaginary parts ($c_{im} < 0$), so the flow is stable. By decreasing c_f for a given k it is possible to reach the point where at least one c_{im} becomes positive and the flow loses stability. The bisection method enables us to find the value of the friction coefficient c_f for which at least one c_{im} is close to zero, while all other c_{im} are negative. This point lies on the "border" between the stability and instability regions of the flow. By repeating the process for different values of the wavenumber k we are able to build a neutral stability curve that is defined as a set of all points in the (k, c_f)-plane for which one c_m has the imaginary part equal to zero, while imaginary parts of all other c_m are negative. The neutral stability curve represents the boundary separating the stability domain (above the curve) from the instability domain (below the curve). The critical value, $c_f^{(c)}$ of the parameter c_f is defined as the coordinate of the highest point of the curve, or $c_f^{(c)} = \max_k(c_f(k))$. The $c_f^{(c)}$ parameter is very important in the linear stability analysis. The flow is stable for all k if the value of c_f is higher than $c_f^{(c)}$, and flow is unstable for some k if $c_f < c_f^{(c)}$.

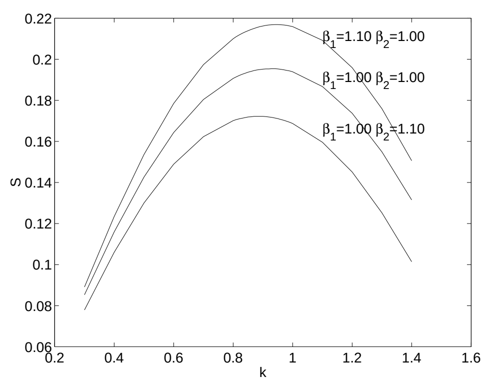

Figure 1. Neutral stability curves versus k for different values of momentum correction coefficients β_1 and β_2.

4. Computational Results

In this section the influence of momentum correction coefficients on the value of $c_f^{(c)}$ parameter is analyzed. The influence is evaluated by solving problem (2.11) for different values of momentum correction coefficients β_1 and β_2, and comparing the critical values, $c_f^{(c)}$, of the parameter c_f. The linear stability results are presented for the hyperbolic secant wake profile in terms of the stability parameter $S = c_f b/h$, where b is the half-width of the wake (see [5]). The values of $S^c = c_f^{(c)} b/h$ have been calculated for the following values of the parameters β_1, and β_2: $\beta_1 = 1.00, 1.05, 1.10$. $\beta_2 = 1.00, 1.05, 1.10$. The value of R of the wake profile is fixed at $R = -0.5$. The parameter N in (3.1) is directly related to the accuracy of computations. We have tried different values of N. It was found that the value $N = 50$ provides sufficient degree of accuracy and, therefore, all numerical results generated in the paper are obtained for the case $N = 50$. The stability curves obtained for various values of momentum correction coefficients β_1 and β_2 are presented in Fig. 1. Each curve represents the boundary between the stability domain (above the curve) and the instability domain (below the curve). The ordinate of the top of the curve corresponds to the critical value of the parameter S.

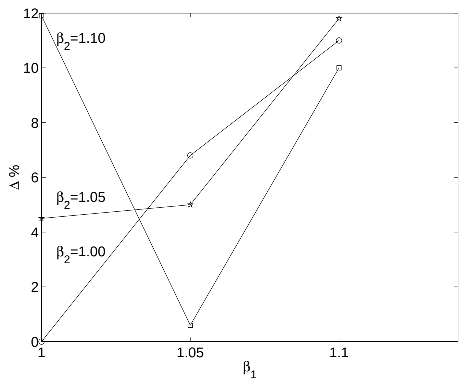

Figure 2. The percentage difference Δ between the values of the S^c for depth-averaged equations ($\beta_1 = 1$, $\beta_2 = 1$) and equations with correction factors ($\beta_1 > 1$,$\beta_2 > 1$).

Fig. 2 presents results of the comparison of the S^c parameter calculated for different values of momentum correction coefficients β_1 and β_2. The results are compared to the values of S^c that are calculated for $\beta_1 = 1$ and $\beta_2 = 1$. The case $\beta_1 = 1$ and $\beta_2 = 1$ corresponds to the approach when the velocity non-uniformity across the vertical coordinate is not taken into account. As it can be seen, for some combinations of the values of β_1 and β_2 the relative error can reach 10%. The real and imaginary parts of the eigenfunction, $\phi(x) = \phi_r(x) + i\phi_i(x)$, are shown in Fig. 3 and Fig. 4 for $R = -0.9$ and $\beta_1 = \beta_2 = 1$. Unfortunately, the values of coefficients β_1 and β_2 for real island wakes are not known. However as the error in determining the S^c parameter may grow with increased values of β_1 (the stability boundary can be underestimated with increase of β_1) it might be important to know the values of β_1 and β_2 for the analyzed shallow flows.

5. Discussion

It is shown in [11] that if the value of the friction coefficient is slightly smaller than the critical value $c_f^{(c)}$ then in the vicinity of the critical point $(k_c, c_f^{(c)})$,

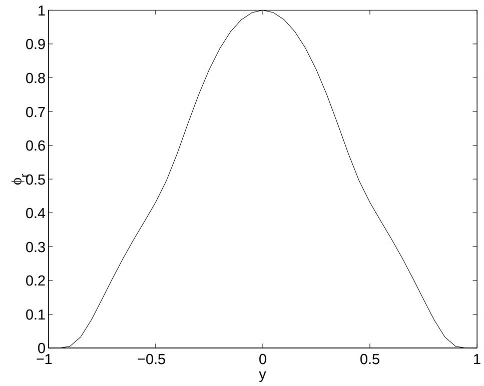

Figure 3. The real part of an eigenfunction obtained at $\beta_1 = 1$, $\beta_2 = 1$ and $R = -0.9$.

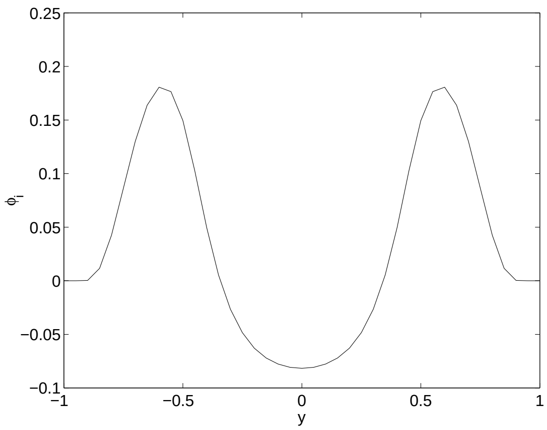

Figure 4. The imaginary part of an eigenfunction obtained at $\beta_1 = 1$, $\beta_2 = 1$ and $R = -0.9$.

where k_c is the critical wavenumber, the evolution of the most unstable mode is governed by the complex Ginzburg-Landau equation of the form

$$\frac{\partial A}{\partial \tau} = \sigma A + \delta \frac{\partial^2 A}{\partial \xi^2} - \mu |A|^2 A, \tag{5.1}$$

where τ and ξ are "slow" time and longitudinal coordinate, respectively, A is the amplitude of the least stable mode and

$$\sigma = \sigma_r + i\sigma_i, \quad \delta = \delta_r + \delta_i, \quad \mu = \mu_r + i\mu_i$$

are complex coefficients. Equation (5.1) is derived in [11] by means of weakly nonlinear theory and explicit formulas for the calculation of the coefficients σ, δ and μ are presented. The coefficients of the Ginzburg-Landau equation are calculated in [10] for different values of β_1 and β_2. The constant μ_r is referred to as the Landau constant in the literature. Calculations performed in [10] showed that for all cases considered the Landau constant is found to be positive. Physically this means that nonlinearities tend to saturate the instability. In other words, the Ginzburg-Landau model with $\mu_r > 0$ shows the existence of another equilibrium state if c_f is slightly smaller than $c_f^{(c)}$ (that is, if the assumptions of the weakly nonlinear theory are satisfied). Note, however, that the Ginzburg-Landau equation should produce meaningful results when the flow is convectively unstable (see [5]). As pointed out by [9], linear (and weakly nonlinear) analyses represent one of the three directions in which two-dimensional coherent structures in shallow water flows are analyzed, that is, stability analysis, experimental investigation, and numerical simulation. In addition, a combination of stability analyses with experimental investigations is found to be quite successful [13] in describing the evolution of shallow mixing layers. The results obtained in the present paper in combination with weakly nonlinear analyses [10, 11] suggest the following approach to the analysis of onset and initial development of coherent structures in shallow flows. For given β_1 and β_2 solve the linear stability problem (2.11) and find the critical values of the parameters of the problem. Next, calculate the coefficients of the Ginzburg-Landau equation (5.1) using the results from [10, 11]. Finally, use the Ginzburg-Landau model to analyze further development of the most unstable mode. The Ginzburg-Landau equation (5.1) is much simpler than the original system which is a definite advantage for numerical simulation. However, a full spatio-temporal analysis of wake flows is required (that is, numerical solution of nonlinear shallow water equations is needed) for complete verification of the Ginzburg-Landau model.

References

[1] D. Chen and G. H. Jirka. Experimental study of plane turbulent wake in a shallow water layer. *Fluid Dynamic Research*, **16**, 11–41, 1995.

[2] D. Chen and G. H. Jirka. Absolute and convective instabilities of plane turbulent wakes in a shallow water layer. *Journal of Fluid Mechanics*, **338**, 157–172, 1997.

[3] M. Van Dyke. *An album of Fluid Motion.* The Parabolic Press, 1982.

[4] M. Ghidaoui and A. A. Kolyshkin. Linear stability analysis of lateral motions in compound open channels. *Journal of Hydraulic Engineering*, **125**, 871–880, 1999.

[5] M. Ghidaoui and A. A. Kolyshkin. Stability analysis of shallow wake flows. *Journal of Fluid Mechanics*, **494**, 355–377, 2003.

[6] M. Ghidaoui, A. A. Kolyshkin and B. C. Yen. Influence of momentum correction coefficients on linear stability analysis of open-channel flows. In: *Proc. of XXVIII IAHR Congress, Theme E5, CD-ROM*, 1999.

[7] P. Holmes, J. L. Lumley and G. Berkooz. *Turbulence, coherent structures, dynamical systems and symmetry.* Cambridge University Press, 1998.

[8] R. G. Ingram and V. H. Chu. Flow around islands in rupert bay: An investigation of the bottom friction effect. *Journal of Geophysical Research*, **92**, 14521–14533, 1987.

[9] J. H. Jirka. Large scale flow structures and mixing processes in shallow flows. *Journal of Hydraulic Research*, **39**, 567–573, 2001.

[10] A. A. Kolyshkin and S. Nazarovs. Calculations of the coefficients of the Ginzburg-Landau equation for shallow water flows. *Scientific Proceedings of the Riga Technical University, Series - Computer Science*, **47**, 48–53, 2005.

[11] A. A. Kolyshkin and S. Nazarovs. Influence of averaging coefficients on weakly nonlinear stability of shallow flows. *IASME Transactions*, **2**, 86–91, 2005.

[12] A. A. Kolyshkin and S. Nazarovs. On the stability of wake flows in shallow water. In: *Proc. of 10th International conference MMA2005*, 2005.

[13] B.C.Van Prooijen and W.S.J. Uijttewaal. A linear approach for the evolution of coherent structures in shallow mixing layers. *Physics of Fluids*, **14**, 4105–4114, 2002.

[14] P. J. Schmid and D. S. Henningson. *Stability and transition in shear flows.* Springer, 2001.

[15] S. A. Socolofsky, C. Carmer and G. H. Jirka. Shallow turbulent wakes: linear stability analysis compared to experimental data. *Shallow flows*, 31–38, 2003. A.A. Balkema Publishers

[16] V. L. Streeter, E. B. Wylie and K. W. Bedford. *Fluid Mechanics.* McGraw Hill, 1998.

[17] D. Tordella and M. Belan. On the domain of validity of the near-parallel combined stability analysis for the 2d intermediate and far bluff body wake. *ZAMM*, **85**, 51–65, 2005.

[18] R. Xia and B. C. Yen. Significance of averaging coefficients in open-channel flow equations. *Journal of Hydraulic Engineering*, **120**, 169–189, 1994.

[19] B.C. Yen. Open-channel flow equations revisited. *Journal of the Engineering Mechanics Division*, **99**, 979–1009, 1973.