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SCALING GROUP OF TRANSFORMATIONS FOR BOUNDARY LAYER STAGNATION-POINT FLOW THROUGH A POROUS MEDIUM TOWARDS A HEATED STRETCHING SHEET

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Abstract. An analysis is performed to investigate the structure of the boundary layer stagnation-point flow and heat transfer of a fluid through a porous medium over a stretching sheet. A scaling group of transformations is applied to get the invariants. Using the invariants, a third and a second order ordinary differential equations corresponding to the momentum and energy equations are obtained respectively. The equations are then solved numerically. It is found that the horizontal velocity increases with the increasing value of the ratio of the free stream velocity (ax) and the stretching velocity (cx). The temperature decreases in this case. At a particular point of the stretching sheet, the fluid velocity decreases or increases with the increase of the permeability of the porous medium when the free stream velocity is less or grater than the stretching velocity.

Key words: scaling group of transformations, stagnation-point flow, porous medium, stretching sheet

Nomenclature

F non-dimensional stream function, F^* variable.
F', F'', F''' first, second and third order derivatives with respect to η.
G absolute invariant defined in $G = x^r\psi^*$.
k permeability of the porous medium, k_1 porosity parameter.
Pr Prandtl number, p, q variables.
T temperature of the fluid, T_w temperature of the wall of the surface.
T_∞ free-stream temperature.
u, v components of velocity in the x and y directions, z variable.

Greek symbols:
α_1, α_2, α_3, α_4, α_5, α_6, α', α'' transformation parameters.
β', β'' transformation parameters, η similarity variable.
Γ Lie-group transformations.
κ the coefficient of thermal diffusivity, μ dynamic viscosity.
ν kinematic viscosity, ψ stream function.
ψ^* variable, ρ density of the fluid.
θ non-dimensional temperature, θ^*,$\bar{\theta}$ variables.
θ', θ'' first and second order derivatives with respect to η.

1. Introduction

The boundary layer equations are especially interesting from a physical point of view because they have the capacity to admit a large number of invariant solutions, i.e. basically closed-form solutions. In the present context, invariant solutions are meant to be a reduction to a simpler equation such as an ordinary differential equation (ODE). Prandtl's boundary layer equations admit more and different symmetry groups. Symmetry groups or simply symmetries are invariant transformations which do not alter the structural form of the equation under investigation (Bluman and Kumei [3]).

The main advantage of the symmetry method is that it can be applied successfully to non-linear differential equations governing the motion of viscous fluid. Lie group analysis was named after Sophius Lie who developed it to find point transformations which map a given differential equation to itself. This method unifies almost all known exact integration techniques for both ordinary and partial differential equations and no ad hoc assumptions or a prior knowledge of the equation under investigation is needed (Kalpakides and Balassas [7]). The differential equation remains invariant under some continuous group of transformations usually known as symmetries of a differential equation. Actually a symmetry group maps any solution to another solution (Koureas et. al.[8]). In case of the scaling group of transformations, the group-invariant solutions are none but the well-known similarity solutions (Pakdemirli and Yurusoy [13]. Similarity solutions are very useful in the sense that they reduce the independent variables of the problem (Ames [2]). In this paper, we apply a special form of Lie group transformations to the problem of stagnation-point flow and heat transfer through a porous medium over a stretching sheet.

The study of hydrodynamic flow and heat transfer through a porous medium towards a stretching sheet becomes much more interesting due to its vast applications on the boundary layer flow control. In certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs and exothermic and / or endothermic chemical reactions and dissociating fluids in the packed-bed reactors, the working fluid heat generation (source) or absorption (sink) effects are important. Representative studies dealing with these effects have been reported by authors such as Gupta and Sridhar [6], Abel and Veena [1]. The heat, mass and momentum transfer in the laminar boundary layer flow on a stretching sheet are important from theoretical as well as practical point of view because of their wider applications to polymer technology and metallurgy. Crane [5] gave an exact similarity solution in closed analytical form for steady boundary layer flow of an incompressible viscous fluid past a stretching elastic plate. MacCormac and Crane [11] studied the steady two-dimensional incompressible boundary layer flow of a Newtonian fluid, caused by the stretching of an elastic flat sheet which moves in its own plane under a uniform stress and the velocity varies linearly with the distance from a fixed point. Chiam [4] investigated two-dimensional steady stagnation-point flow of an incompressible viscous fluid towards a stretching surface. Roy Mahapatra and Gupta [9, 10] studied the MHD stagnation-point flow and heat transfer in case of stagnation-point flow. Recently, Nazar et. al. [12] studied the unsteady boundary layer flow in the region of stagnation-point on a stretching sheet.

The purpose of this paper is to investigate the steady two-dimensional stagnation-point flow of an incompressible viscous fluid through a porous medium towards a stretching surface. The temperature distribution is obtained when the stretching surface is held at a constant temperature. The momentum and the thermal boundary layer equations are solved using shooting method. The results, thus obtained, are then presented graphically and analysed.

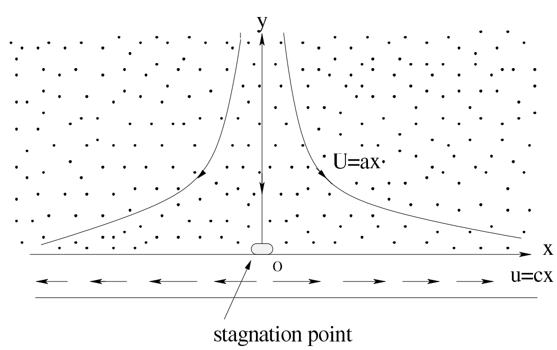

Figure 1. A sketch of the physical problem.

2. Equations of Motion

We consider the two-dimensional steady flow of an incompressible viscous liquid through a porous medium near a stagnation point at a surface coinciding with the plane $y = 0$, the flow being confined to $y > 0$. We introduce two equal and opposite forces along the x-axis so that the wall is stretched keeping the origin fixed (Fig.1). The boundary layer equations for steady two-dimensional stagnation-point flow through a porous medium (highly permeable) over the stretching surface (with the application of Darcy's law) are given as

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0, \\ u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = U\dfrac{\partial U}{\partial x} + \nu\dfrac{\partial^2 u}{\partial y^2} + \dfrac{\nu}{k}(U-u). \end{cases} \tag{2.1}$$

In equation (2.1), $U(x)$ stands for the stagnation-point velocity in the inviscid free stream, u and v are the components of velocity respectively in the x and y directions, k is the permeability of the porous medium, μ is the coefficient of fluid viscosity, ρ is the fluid density, $\nu = \mu/\rho$ is the kinematic viscosity.

By using the boundary layer approximations and neglecting viscous dissipation, the equation for temperature T is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} \tag{2.2}$$

where κ is the coefficient of thermal diffusivity of the fluid.

2.1. Boundary conditions

The appropriate boundary conditions for the above problem are given by

$$u = cx, \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0, \tag{2.3}$$

$$u \to U(x) = ax, \; T \to T_\infty \quad \text{as} \quad y \to \infty. \tag{2.4}$$

Here $c(>0)$ and $a(>0)$ are constants, T_w is the uniform wall temperature, T_∞ is the free stream temperature, T_w and T_∞ are also constants with $T_w > T_\infty$.

2.2. Method of solution

We now introduce the following relations for u, v and θ as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{2.5}$$

where ψ is the stream function. Using the relation (2.5) in the boundary layer equation (2.1) and in the energy equation (2.2) we get the following equations

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = U\frac{\partial U}{\partial x} + \nu\frac{\partial^3 \psi}{\partial y^3} + \frac{\nu}{k}(U - \frac{\partial \psi}{\partial y}) \tag{2.6}$$

and

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \kappa\frac{\partial^2 \theta}{\partial y^2}. \tag{2.7}$$

The boundary conditions (2.3) and (2.4) then become

$$\frac{\partial \psi}{\partial y} = cx, \quad \frac{\partial \psi}{\partial x} = 0, \quad \theta = 1 \quad \text{at} \quad y = 0,$$

$$\frac{\partial \psi}{\partial y} \to U(x) = ax, \quad \theta \to 0 \quad \text{as} \quad y \to \infty.$$

2.3. Scaling group of transformations

We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations (see Tapanidis *et al.* [14]),

$$\begin{aligned} \Gamma : x^* = xe^{\epsilon\alpha_1}, \quad & y^* = ye^{\epsilon\alpha_2}, \quad \psi^* = \psi e^{\epsilon\alpha_3}, \\ u^* = ue^{\epsilon\alpha_4}, \quad v^* = ve^{\epsilon\alpha_5}, \quad & U^* = Ue^{\epsilon\alpha_6}, \quad \theta^* = \theta e^{\epsilon\alpha_7}. \end{aligned} \tag{2.8}$$

Equation (2.8) may be considered as a point-transformation which transforms co-ordinates $(x, y, \psi, u, v, U, \theta)$ to the co-ordinates $(x^*, y^*, \psi^*, u^*, v^*, U^*, \theta^*)$.

Substituting (2.8) in (2.6) and (2.7) we get,

$$\begin{cases} e^{\epsilon(\alpha_1+2\alpha_2-2\alpha_3)}\Big(\dfrac{\partial\psi^*}{\partial y^*}\dfrac{\partial^2\psi^*}{\partial x^*\partial y^*} - \dfrac{\partial\psi^*}{\partial x^*}\dfrac{\partial^2\psi^*}{\partial y^{*2}}\Big) = e^{\epsilon(\alpha_1-2\alpha_6)}U^*\dfrac{\partial U^*}{\partial x^*} \\ \qquad +\nu e^{\epsilon(3\alpha_2-\alpha_3)}\dfrac{\partial^3\psi^*}{\partial y^*3} + \dfrac{\nu}{k}e^{-\epsilon\alpha_6}U^* - \dfrac{\nu}{k}e^{\epsilon(\alpha_2-\alpha_3)}\dfrac{\partial\psi^*}{\partial y^*}, \\ e^{\epsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_7)}\Big(\dfrac{\partial\psi^*}{\partial y^*}\dfrac{\partial\theta^*}{\partial x^*} - \dfrac{\partial\psi^*}{\partial x^*}\dfrac{\partial\theta^*}{\partial y^*}\Big) = \kappa e^{\epsilon(2\alpha_2-\alpha_7)}\dfrac{\partial^2\theta^*}{\partial y^{*2}}. \end{cases} \tag{2.9}$$

The system will remain invariant under the group of transformations Γ, so we would have the following relations among the transformation parameters, namely

$$\begin{aligned} &\alpha_1 + 2\alpha_2 - 2\alpha_3 = \alpha_1 - 2\alpha_6 = 3\alpha_2 - \alpha_3 = -\alpha_6 = \alpha_2 - \alpha_3 \\ &\alpha_1 + \alpha_2 - \alpha_3 - \alpha_7 = 2\alpha_2 - \alpha_7. \end{aligned} \tag{2.10}$$

From the first relation in (2.10) we get

$$\alpha_2 - \alpha_3 + \alpha_6 = 0.$$

The third relation gives the value $\alpha_2 = 0$. From $\alpha_2 - \alpha_3 + \alpha_6 = 0$ we get, $\alpha_3 = \alpha_6$ (since $\alpha_2 = 0$). Again from $\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3$ we get, $\alpha_1 = \alpha_3$. In view of these, the boundary conditions become

$$\begin{aligned} &\frac{\partial\psi^*}{\partial y^*} = cx^*, \quad \frac{\partial\psi^*}{\partial x^*} = 0, \theta^* = 1 \quad \text{at} \quad y^* = 0, \\ &\frac{\partial\psi^*}{\partial y^*} \to U^* = ax^*, \quad \theta^* \to 0 \quad \text{as} \quad y^* \to \infty \end{aligned}$$

with the additional conditions $\alpha_1 = \alpha_3 = \alpha_4 = \alpha_6$, $\alpha_2 = \alpha_5 = \alpha_7 = 0$. Thus the set Γ reduces to a one parameter group of transformations as

$$\begin{aligned} x^* = xe^{\epsilon\alpha_1}, \quad & y^* = y, \psi^* = \psi e^{\epsilon\alpha_1}, \quad u^* = ue^{\epsilon\alpha_1}, \\ v^* = v, \quad & U^* = Ue^{\epsilon\alpha_1}, \quad \theta^* = \theta. \end{aligned} \tag{2.11}$$

2.4. Absolute invariants

A variable or function which retains same structural form in a particular mathematical transformation is known as an invariant. If a variable or function retains its structural form in all mathematical transformations then it is called an absolute invariant. In this paper, absolute invariants are nothing but the similarity variables and the similarity solutions.

First we derive an absolute invariant which is a function of the dependent variable, namely $\eta = yx^s$. For this purpose, we write

$$x^* = Bx, \quad B = e^{\epsilon\alpha_1}, \quad y^* = B^{\frac{\alpha_2}{\alpha_1}} y, \quad \psi^* = B^{\frac{\alpha_3}{\alpha_1}}\psi, \quad U^* = UB^{\frac{\alpha_6}{\alpha_1}}.$$

To establish $y^* x^{*s} = yx^s$, we have

$$y^* x^{*s} = yB^{\frac{\alpha_2}{\alpha_1}} B^s x^s = yx^s B^{s+\frac{\alpha_2}{\alpha_1}}.$$

Putting $s + \dfrac{\alpha_2}{\alpha_1} = 0$ we get, $y^* x^{*s} = yx^s$. Since $\alpha_2 = 0$ so $s = 0$ and we get $\eta = y^*$. Thus we obtain

$$\eta = y^* \tag{2.12}$$

as an absolute invariant.

We now find a second absolute invariant G, which involves the dependent variable ψ. Let us assume that $G = x^r \psi$. We will find r such that $x^r\psi = x^{*r}\psi^*$, then

$$x^{*r}\psi^* = B^r x^r B^{\frac{\alpha_3}{\alpha_1}} \psi = B^{r+\frac{\alpha_3}{\alpha_1}} x^r \psi.$$

Now by putting, $r + \dfrac{\alpha_3}{\alpha_1} = 0$ we get, $r = -\dfrac{\alpha_3}{\alpha_1} = -1$ (since $\alpha_1 = \alpha_3$). Thus, we get the second absolute invariant G as $G = x^{*-1}\psi^*$. Putting $G = F(\eta)$ we can write

$$\psi^* = x^* F(\eta). \tag{2.13}$$

We also have $\theta^* = \theta(\eta)$.

In view of relations (2.12) and (2.13), the equations (2.9) become

$$\begin{cases} F'^2 - FF'' = a^2 + \nu F''' + \dfrac{\nu}{k}(a - F'), \\ F\theta' + \kappa\theta'' = 0. \end{cases} \tag{2.14}$$

The boundary conditions are transformed to

$$\begin{aligned} &F'(\eta) = c, \quad F(\eta) = 0 \quad \text{and} \quad \theta(\eta) = 1 \quad \text{at} \quad \eta = 0. \\ &F'(\eta) \to a, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty. \end{aligned}$$

Again we introduce the following transformations for η , F and θ in equations (2.14):

$$\eta = \nu^\alpha c^\beta \eta^*, \quad F = \nu^{\alpha'} c^{\beta'} F^*, \quad \theta = \nu^{\alpha''} c^{\beta''} \bar{\theta}.$$

Taking $F^* = f$ and $\bar{\theta} = \theta$ the equations (2.14) finally take the following form:

$$\begin{cases} f''' + ff'' - f'^2 + \frac{a^2}{c^2} + k_1(\frac{a}{c} - f') = 0, \\ \theta'' + Prf\theta' = 0, \end{cases} \tag{2.15}$$

where $k_1 = \dfrac{\nu}{kc}$ is the permeability parameter of the porous medium and $Pr = \dfrac{\nu}{\kappa}$ is the Prandtl number. The boundary conditions take the form

$$f' = 1, \quad f = 0, \quad \theta = 1 \quad \text{at} \quad \eta^* = 0$$
$$f' \to \frac{a}{c}, \quad \theta \to 0 \quad \text{as} \quad \eta^* \to \infty.$$

3. Numerical Method for Solution

The above system of equations (2.15) along with boundary conditions are solved by converting them to an initial value problem. We set

$$\begin{cases} f' = z, \quad z' = p, \\ p' = z^2 - fp - \dfrac{a^2}{c^2} - k_1(\dfrac{a}{c} - z), \\ \theta' = q, \quad q' = -Prfq \end{cases} \tag{3.1}$$

with the initial conditions

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1.$$

In order to integrate (3.1) as an initial value problem we require to give values for $p(0) = f''(0)$ and $q(0) = \theta'(0)$ but no such values are given on the boundary. We use the shooting method. Suitable initial values for $f''(0)$ and $\theta'(0)$ are chosen and then integration is carried out. We compare the calculated values for f' and θ at the end of time integration interval $\eta = 5$ with the given boundary conditions $f'(5) = \dfrac{a}{c}$ and $\theta(5) = 0$ and adjust the estimated values, $f''(0)$ and $\theta'(0)$, to give a better approximation for the solution.

We take a series of values for $f''(0)$ and $\theta'(0)$ and apply the fourth order classical Runge-Kutta method with step-size $h = 0.01$. The above procedure is repeated until we get the results up to the desired degree of accuracy, 10^{-5}.

4. Results and Discussions

Computation through employed numerical scheme has been carried out for various values of the parameters such as parameter a/c, permeability parameter k_1 and Prandtl number Pr. For illustrations of the results, numerical values are plotted in the figures.

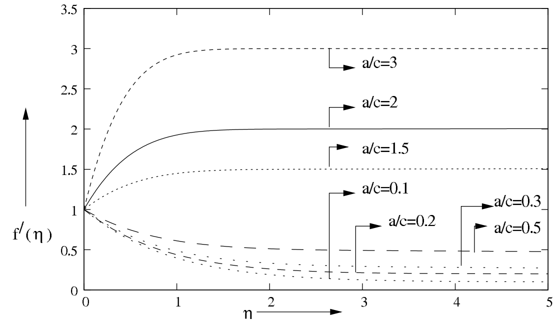

Figure 2. Variation of horizontal velocity $f'(\eta)$ with η for several values of a/c with $Pr = 0.05, k_1 = 0$.

First, we present the result for the variation of the parameter a/c when the sheet is not porous, i.e. the parameter $k_1 = 0$. In Fig.2, horizontal velocity profiles are shown for different values of a/c. Two set of values for a/c, i.e. $a/c < 1$ and $a/c > 1$ are considered. It is seen that the horizontal velocity increases with the increase of non-zero values of a/c. It is evident from this figure that when $a/c > 1$, the flow has a boundary layer structure and the thickness of the boundary layer decreases with the increase in a/c. It can be explained as follows. For fixed value of c, corresponding to the stretching of the surface, increase in a in relation to c (such that $a/c > 1$) implies increase in straining motion near the stagnation region. Due to this reason the acceleration of the external stream is increased and this leads to thinning of the boundary layer. On the other hand when $a/c < 1$, the flow has an inverted boundary layer structure. In this case, the stretching velocity (cx) of the surface exceeds the velocity (ax) of the external stream. It is to be noted that no boundary layer is formed when $a/c = 1$.

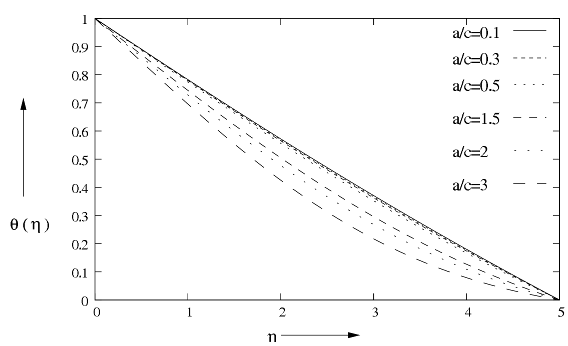

Figure 3. Variation of temperature $\theta(\eta)$ with η for several values of a/c with $Pr = 0.05, k_1 = 0$.

Fig.3 represents the temperature profiles for different values of

$$\frac{a}{c} = 0.1, \;\; 0.3, \;\; 0.5 < 1, \qquad \frac{a}{c} = 1.5, \;\; 2, \;\; 3 > 1.$$

For all values of a/c considered, θ is found to decrease with the increase of η. There is no significant change in the rate of decrease of θ for the different values of a/c when $a/c < 1$. Temperature at a point on the sheet decreases significantly with the increase in a/c.

Fluid flow and heat transfer towards a porous stretching sheet have an important bearing on several technological processes. Some metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. The rate of cooling can be controlled and final product of desired characteristics can be achieved if strips are drawn through porous media. With this motivation we studied the steady two-dimensional stagnation-point flow of an incompressible fluid in presence of porous medium towards a stretching surface.

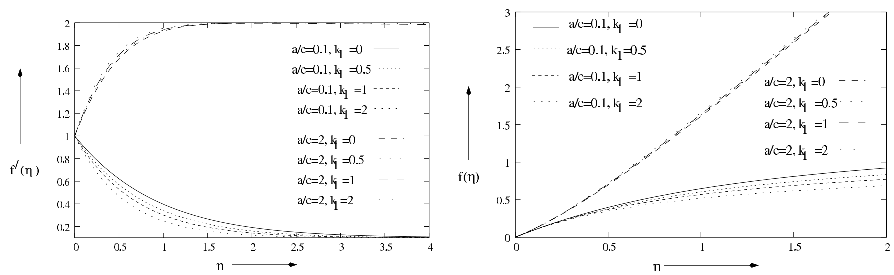

Figure 4. (a)Variation of horizontal velocity $f'(\eta)$ with η for several values of k_1 and a/c with $Pr = 0.05$;(b) Variation of transverse velocity $f(\eta)$ with η for several values of k_1 and a/c with $Pr = 0.05$.

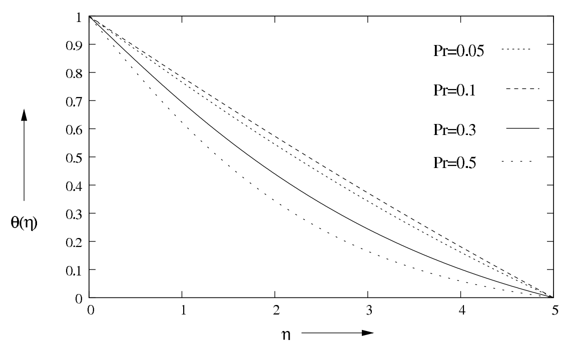

Figure 5. Variation of temperature $\theta(\eta)$ with η for several values of Pr with $a/c = 0.1, k_1 = 0.1$

Figs.4(a) and 4(b) are the graphical representation of horizontal velocity profile $f'(\eta)$ and transverse velocity profile $f(\eta)$ for the different values of the permeability parameter k_1 of the porous medium ($k_1 = 0.0, 0.5, 1, 2$) when the Prandtl number Pr is 0.05 for two different set of values of a/c lying in $0.1 < 1$ and $2 > 1$. It is found that for $a/c < 1$, the horizontal velocity $f'(\eta)$ decreases with the increase of k_1 but it increases with the increase of k_1 when $a/c > 1$. From Fig.4(b) it is very clear that the transverse velocity decreases with the increase of k_1 when $a/c (= 0.1 < 1)$ ($Pr = 0.05$) but k_1 has no significant effect on the transverse velocity when $a/c (= 2 > 1)$.

Fig.5 shows the effects of the Prandtl number (Pr) on the temperature $\theta(\eta)$ for fixed values of $a/c = 0.1, k_1 = 0.1$. As anticipated, the thermal boundary layer thickness decreases with increasing the Prandtl number (i.e. with the decreasing thermal diffusivity). It is clear from this figure that the temperature at a point decreases with increase in the Prandtl number Pr but the increase of Pr has no such effect on the horizontal velocity.

Table 1. Values of $f''(0)$ for several values of a/c and k_1 with $Pr = 0.05$.

k_1	$a/c \to 0.1$	0.5	2.0	3.0
0.0	-0.9601	-0.6499	1.9991	4.5011
0.1	-0.8910	-0.5011	2.0101	4.8011
0.5	-0.8001	-0.3711	2.1102	4.9102
1.0	-0.7191	-0.3402	2.3905	4.9691
1.5	-0.5692	-0.3112	2.7201	4.9901
2.0	-0.5502	-0.2901	3.1511	5.5010

Finally, we compute the dimensionless shear stress at the wall for various values of a/c and k_1. The values of $f''(0)$ are given in the Table1. Our computed results agree excellently with the results of Nazar et. al.[12] in steady case with $k_1 = 0$. From this table, it is very clear that the numerical value of wall shear stress decreases with the increase in k_1, for a fixed value of a/c when $a/c < 1$ (values of $f''(0)$ are negative in this case) and increases with the increasing k_1 for $a/c > 1$ (values of $f''(0)$ are positive). On the other hand, for a fixed value of k_1, the wall shear stress decreases with the increase in a/c provided $a/c < 1$ but increases with increase in a/c when the values of a/c are greater than 1.

5. Conclusion

Similarity solution of a steady boundary layer flow in the stagnation-point region on a stretching sheet embedded in a porous medium has been obtained by using scaling group of transformations. The results pertaining to the present study indicate that the flow has a boundary layer structure when $a/c > 1$ and when $a/c < 1$, the flow has an inverted boundary layer structure. The effect of porosity parameter on a viscous incompressible liquid is to suppress the velocity field when $a/c < 1$. This in turn causes the enhancement of the velocity field when $a/c > 1$. The temperature at a point is found to decrease with the increase in Pr. The porosity parameter plays a significant role on the wall shear stress.

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