Supplementary information

DEVELOPMENT OF THE LIFE-CYCLE ECONOMIC AND ENVIRONMENTAL ASSESSMENT MODEL FOR ESTABLISHING THE OPTIMAL IMPLEMENTATION STRATEGY OF THE ROOFTOP PHOTOVOLTAIC SYSTEM

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Technological and Economic Development of Economy
https://doi.org/10.3846/tede.2021.15704

Supplement A. Equivalent transformation of the objective function

Case 1-1: $M_j \le FT \le T_w$

Using $F+(1-F)\beta = 1-(1-F)(1-\beta)$ and $\pi = p + c_g - c$, Eq. (7) can be rewritten as below:

$$ATP_{11}^{(j)}(F,T) = pD - \left\{ \begin{array}{l} \dfrac{A}{T} + cD + \pi D(1-F)(1-\beta) + \dfrac{c_b \beta D (1-F)^2 T}{2} + \dfrac{h_o D F^2 T}{2} - \dfrac{pI_e D M_j^2}{2T} \\ -pI_e(1-F)D\beta M_j + \dfrac{cI_c D\left(FT - M_j\right)^2}{2T} \end{array} \right\}. \quad (A1)$$

From Eq. (A1), since pD is a constant, $ATP_{11}^{(j)}(F,T)$ is maximized by minimizing the expression inside the bracket, which is

$$\begin{aligned} ATC_{11}^{(j)}(F,T) = {} & \frac{A}{T} + cD + \pi D(1-F)(1-\beta) + \frac{c_b \beta D (1-F)^2 T}{2} + \frac{h_o D F^2 T}{2} - \frac{pI_e D M_j^2}{2T} \\ & -pI_e(1-F)D\beta M_j + \frac{cI_c D\left(FT - M_j\right)^2}{2T} \end{aligned}. \quad (A2)$$

Further, making some algebraic manipulation, Eq. (A2) can be rearranged to Eq. (A3),

$$\begin{aligned} ATC_{11}^{(j)}(F,T) = {} & \underbrace{\frac{D}{2}\left(c_b\beta + h_o + cI_c\right)}_{\psi_{111}} F^2 T - \underbrace{c_b \beta D}_{\psi_{112}} FT - \underbrace{\left(\pi D(1-\beta) + \left(cI_c - \beta p I_e\right) M_j D\right)}_{\psi_{113}} F \\ & + \underbrace{\frac{c_b \beta D}{2}}_{\psi_{114}} T + \underbrace{\left(A + \frac{\left(cI_c - pI_e\right) D M_j^2}{2} \right)}_{\psi_{115}} \frac{1}{T} + \underbrace{cD + \pi D(1-\beta) - pI_e \beta D M_j}_{\psi_{116}} \end{aligned}. \quad (A3)$$

Case 1-2: $FT \le M_j \le T_w$ **or** $FT \le T_w \le M_j$

Similar to the treatment method of Case 1-1, Eq. (8) can be rewritten as below:

$$ATP_{12}^{(j)}(F,T) = pD - \left\{ \begin{array}{l} \dfrac{A}{T} + cD + \pi D(1-F)(1-\beta) + \dfrac{c_b \beta D (1-F)^2 T}{2} + \dfrac{h_o D F^2 T}{2} \\ -pI_e D\left[F\left(M_j - FT/2\right) + (1-F)\beta M_j\right] \end{array} \right\}. \quad \text{(A4)}$$

Maximizing Eq. (A4) is equivalent to minimizing the following function

$$\begin{array}{l} ATC_{12}^{(j)}(F,T) = \underbrace{\dfrac{D}{2}(c_b\beta + h_o + pI_e)}_{\psi_{121}} F^2 T - \underbrace{c_b \beta D}_{\psi_{122}} FT - \underbrace{\left(\pi D(1-\beta) + (1-\beta) pI_e M_j D\right)}_{\psi_{123}} F \\ + \underbrace{\dfrac{c_b \beta D}{2}}_{\psi_{124}} T + \underbrace{A}_{\psi_{125}} \dfrac{1}{T} + \underbrace{cD + \pi D(1-\beta) - pI_e \beta D M_j}_{\psi_{126}} \end{array}. \quad \text{(A5)}$$

Case 2-1: $M_j \le T_w \le FT$ or $T_w \le M_j \le FT$

Similarly, Eq. (9) can be rewritten as follows:

$$ATP_{21}^{(j)}(F,T) = pD - \left\{ \begin{array}{l} \dfrac{A}{T} + cD + \pi D(1-F)(1-\beta) + \dfrac{c_b \beta D (1-F)^2 T}{2} + \dfrac{h_r (FDT - W)^2}{2DT} \\ + \dfrac{h_o (2DFT - W) W}{2DT} - pI_e D\left[\dfrac{M_j^2}{2T} + (1-F)\beta M_j\right] + \dfrac{cI_c D (FT - M_j)^2}{2T} \end{array} \right\}. \quad \text{(A6)}$$

Maximizing Eq. (A6) is equivalent to minimizing the following function

$$\begin{array}{l} ATC_{21}^{(j)}(F,T) = \underbrace{\dfrac{D}{2}(c_b\beta + h_r + c_j I_c)}_{\psi_{211}} F^2 T - \underbrace{c_b \beta D}_{\psi_{212}} FT - \underbrace{\left(\pi D(1-\beta) + (h_r - h_o) W + (cI_c - \beta p I_e) D M_j\right)}_{\psi_{213}} F \\ + \underbrace{\dfrac{c_b \beta D}{2}}_{\psi_{214}} T + \underbrace{\left[A + \dfrac{(h_r - h_o) W^2}{2D} + \dfrac{(cI_c - pI_e) D M_j^2}{2}\right]}_{\psi_{215}} \dfrac{1}{T} + \underbrace{cD + \pi D(1-\beta) - pI_e \beta D M_j}_{\psi_{216}} \end{array}. \quad \text{(A7)}$$

Case 2-2: $T_w \le FT \le M_j$

Again, Eq. (10) can be rewritten as below:

$$ATP_{22}^{(j)}(F,T) = pD - \left\{ \begin{array}{l} \dfrac{A}{T} + cD + \pi D(1-F)(1-\beta) + \dfrac{c_b \beta D (1-F)^2 T}{2} + \dfrac{h_r (FDT - W)^2}{2DT} \\ + \dfrac{h_o (2DFT - W) W}{2DT} - pI_e D\left[F\left(M_j - FT/2\right) + (1-F)\beta M_j\right] \end{array} \right\}. \quad \text{(A8)}$$

Maximizing Eq. (A8) is equivalent to minimizing the following function

$$ATC_{22}^{(j)}(F,T)=\underbrace{\frac{D}{2}\left(c_b\beta+h_r+pI_e\right)}_{\psi_{221}}F^2T-\underbrace{c_b\beta D}_{\psi_{222}}FT-\underbrace{\left(\pi D(1-\beta)+\left(h_r-h_o\right)W+(1-\beta)pI_eDM_j\right)}_{\psi_{223}}F$$
$$+\underbrace{\frac{c_b\beta D}{2}}_{\psi_{224}}T+\underbrace{\left[A+\frac{\left(h_r-h_o\right)W^2}{2D}\right]}_{\psi_{225}}\frac{1}{T}+\underbrace{cD+\pi D(1-\beta)-pI_e\beta DM_j}_{\psi_{226}}. \tag{A9}$$

***Supplement B*. Find the optimal values when $\psi_{115}\le 0$ and $\psi_{215}\le 0$**

For Case 1-1, if $\psi_{115}\le 0$, we have $\theta_{11}(F)=\psi_{111}F^2-\psi_{112}F+\psi_{114}>0$, which always holds for any $F\in[0,1]$. So if $\psi_{115}\le 0$, then

$$ATC_{11}'^{(j)}(F,T)=\psi_{111}F^2-\psi_{112}F+\psi_{114}-\frac{\psi_{115}}{T^2}>0\,. \tag{B1}$$

From Eq. (B1), we learn that $ATC_{11}^{(j)}(F,T)$ is a strictly increasing function of *T*. Therefore, $ATC_{11}^{(j)}(F,T)$ reaches its minimum at $T=\frac{M_j}{F}$. Then substituting $T=\frac{M_j}{F}$ into Eq. (11) lead to

$$ATC_{11}^{(j)}\left(F,\frac{M_j}{F}\right)=\psi_{111}FM_j-\psi_{112}M_j-\psi_{113}F+\psi_{114}\frac{M_j}{F}+\frac{\psi_{115}F}{M_j}+\psi_{116}\,. \tag{B2}$$

Taking the first and second derivatives of Eq. (B2) with respect to *F*, we have

$$ATC_{11}'^{(j)}\left(F,\frac{M_j}{F}\right)=\psi_{111}M_j-\psi_{113}-\psi_{114}\frac{M_j}{F^2}+\frac{\psi_{115}}{M_j}\,; \tag{B3}$$

$$ATC_{11}''^{(j)}\left(F,\frac{M_j}{F}\right)=\frac{2M_j\psi_{114}}{F^3}>0\,. \tag{B4}$$

Clearly, $ATC_{11}^{(j)}(F,T)$ is a strictly convex function of *F*. Setting $ATC_{11}'^{(j)}\left(F,\frac{M_j}{F}\right)=0$ yields

$$F_{11}'=\sqrt{\frac{\psi_{114}M_j^2}{\psi_{111}M_j^2-\psi_{113}M_j+\psi_{115}}}\,. \tag{B5}$$

Therefore, from Eq. (B5), if F_{11}' is feasible, the optimal solution in this case is $\left(T_{11},F_{11}\right)=\left(\frac{M_j}{F_{11}'},F_{11}'\right)$. Otherwise, the optimal solution is $\left(T_{11},F_{11}\right)=\left(M_j,1\right)$.

Similarly, for Case 2-1, if $\psi_{215}\le 0$, following the same steps used in Case 1-1 to develop F_{21}', we have

$$F_{21}'=\sqrt{\frac{\psi_{214}\left(\max\left\{T_w,M_j\right\}\right)^2}{\psi_{211}\left(\max\left\{T_w,M_j\right\}\right)^2-\psi_{213}\max\left\{T_w,M_j\right\}+\psi_{215}}}\,. \tag{B6}$$

For Eq. (B6), if F'_{21} is feasible, the optimal solution in this case is $\left(T_{21},F_{21}\right)=\left(\dfrac{\max\left\{T_w,M_j\right\}}{F'_{21}},F'_{21}\right)$. Otherwise, the optimal solution is $\left(T_{11},F_{11}\right)=\left(\max\left\{T_w,M_j\right\},1\right)$.

***Supplement C.* Find the roots** $\left(F_{11},T_{11}\right)$, $\left(F_{12},T_{12}\right)$, $\left(F_{21},T_{21}\right)$ and $\left(F_{22},T_{22}\right)$.

Case 1-1: $M_j \le FT \le T_w$

From Eq. (11), differentiating $ATC_{11}^{(j)}\left(F,T\right)$ with respect to F and T, we have

$$\frac{\partial ATC_{11}^{(j)}\left(F,T\right)}{\partial F}=2\psi_{111}FT-\psi_{112}T-\psi_{113}\ \rightarrow\ F=\frac{\psi_{112}T+\psi_{113}}{2\psi_{111}T}; \tag{C1}$$

$$\frac{\partial ATC_{11}^{(j)}\left(F,T\right)}{\partial T}=\psi_{111}F^2-\psi_{112}F+\psi_{114}-\frac{\psi_{115}}{T^2}\ \rightarrow\ T^2=\frac{\psi_{115}}{\psi_{111}F^2-\psi_{112}F+\psi_{114}}. \tag{C2}$$

After some algebra, we have

$$T_{11}=\sqrt{\frac{4\psi_{111}\psi_{115}-\psi_{113}^2}{4\psi_{111}\psi_{114}-\psi_{112}^2}}; \tag{C3}$$

$$F_{11}=\frac{\psi_{112}}{2\psi_{111}}+\frac{\psi_{113}}{2\psi_{111}}\sqrt{\frac{4\psi_{111}\psi_{114}-\psi_{112}^2}{4\psi_{111}\psi_{115}-\psi_{113}^2}}. \tag{C4}$$

Similarly, for the other cases, the roots $\left(F_{12},T_{12}\right)$, $\left(F_{21},T_{21}\right)$ and $\left(F_{22},T_{22}\right)$ can be obtained easily.

***Supplement D.* Find the optimal values of $T^{\#}$ when F = 1**

Case 1-1: $M_j \le FT \le T_w$

Substituting F_{11} = 1 into Eq. (10) leads to

$$ATC_{11}^{(j)}\left(1,T\right)=\psi_{111}T-\psi_{112}T-\psi_{113}+\psi_{114}T+\frac{\psi_{115}}{T}+\psi_{116}. \tag{D1}$$

Taking the first and second derivatives of Eq. (D1) with respect to T, we have

$$\frac{dATC_{11}^{(j)}\left(1,T\right)}{dT}=\psi_{111}-\psi_{112}+\psi_{114}-\frac{\psi_{115}}{T^2}; \tag{D2}$$

$$\frac{dATC_{11}^{(j)}\left(1,T\right)}{dT}=\frac{2\psi_{115}}{T^3}. \tag{D3}$$

Obviously, if ψ_{115} > 0, then $ATC_{11}^{(j)}\left(1,T\right)$ is a strictly convex function of T. Setting $ATC_{11}^{\prime(j)}\left(1,T\right)=0$ yields

$$T_{11}^{\#}=\sqrt{\frac{\psi_{115}}{\psi_{111}-\psi_{112}+\psi_{114}}}. \tag{D4}$$

Analogously, for the other cases, $T^{\#}$ also can be found when F_{12} = 1, F_{21} = 1 and F_{22} = 1,

$$T_{12}^{\#}=\sqrt{\frac{\psi_{125}}{\psi_{121}-\psi_{122}+\psi_{124}}}; \tag{D5}$$

$$T_{21}^{\#}=\sqrt{\frac{\psi_{215}}{\psi_{211}-\psi_{212}+\psi_{214}}}\,; \tag{D6}$$

$$T_{22}^{\#}=\sqrt{\frac{\psi_{225}}{\psi_{221}-\psi_{222}+\psi_{224}}}\,. \tag{D7}$$

Supplement E. **Find the optimal values of F' and T'**

For the solutions of F_{11} and T_{11} obtained for Case 1-1, if the relationship $F_{11}T_{11} < M_j$ is established, it shows that the optimal values will be obtained on the boundary point. Thus, we may set $T=\frac{M_j}{F_{11}}$ and then substitute it into Eq. (11), which leads to

$$ATC_{11}^{(j)}\left(F_{11},\frac{M_j}{F_{11}}\right)=\psi_{111}F_{11}^2\frac{M_j}{F_{11}}-\psi_{112}F_{11}\frac{M_j}{F_{11}}-\psi_{113}F_{11}+\psi_{114}\frac{M_j}{F_{11}}+\frac{\psi_{115}F_{11}}{M_j}+\psi_{116}\,. \tag{E1}$$

Taking the first and second derivatives of Eq. (E1) with respect to F_{11}, we have

$$\frac{dATC_{11}^{(j)}\left(F_{11},M_j/F_{11}\right)}{dF_{11}}=\psi_{111}M_j-\psi_{113}-\psi_{114}\frac{M_j}{F_{11}{}^2}+\frac{\psi_{115}}{M_j}\,; \tag{E2}$$

$$\frac{d^2ATC_{11}^{(j)}\left(F_{11},M/F_{11}\right)}{dF_{11}^2}=\frac{2\psi_{114}}{F_{11}{}^2}M_j>0\,. \tag{E3}$$

From Eq. (E3), $ATC_{11}^{(j)}\left(F_{11},\frac{M_j}{F_{11}}\right)$ is a strictly convex function of F_{11}.

Setting $ATC_{11}'^{(j)}\left(F_{11},\frac{M_j}{F_{11}}\right)=0$ yields

$$F_{11}'=\sqrt{\frac{\psi_{114}M_j^2}{\psi_{111}M_j^2-\psi_{113}M_j+\psi_{115}}}\,. \tag{E4}$$

Noticing that, if F_{11}' is feasible, the optimal solution in this case is $\left(T_{11},F_{11}\right)=\left(\frac{M_j}{F_{11}'},F_{11}'\right)$, otherwise the optimal solution is $\left(T_{11},F_{11}\right)=\left(M_j,1\right)$.

In addition, if the relationship $F_{11}T_{11} > T_w$ is established, use the same approach to develop F_{11}', $F_{11}'=\sqrt{\frac{\psi_{114}T_w^2}{\psi_{111}T_w^2-\psi_{113}T_w+\psi_{115}}}$.

In the same way, we can analyze Case 1-2, Case 2-1 and Case 2-2. The specific computational results are summarized in Table 2.

Supplement F. **Find the optimal values of T'' and F'' when optimal order quantity $Q_j\notin\left[q_j,q_{j+1}\right)$**

If $Q_j\notin\left[q_j,q_{j+1}\right)$, there are two situations:

(1) if $Q_j\geq q_{j+1}$, the optimal solution does not exist and then the retailer needs to adjust the order quantity;

(2) if $Q_j < q_j$, the optimal values will be obtained at point $T=\frac{q_j}{D\left[\left(1-F\right)\beta+F\right]}$.

Based on the analysis above, we only need to discuss the case of $Q_j < q_j$.

First, for Case 1-1, substituting $T=\frac{q_j}{D\left[(1-F)\beta+F\right]}=\frac{q_j}{D\left[(1-\beta)F+\beta\right]}$ into Eq. (A2) leads to

$$\begin{aligned}ATC_{11}^{(j)}(F)=&\frac{\left[2AD+(cI_c-pI_e)D^2M_j^2\right]\left[(1-\beta)F+\beta\right]}{2q_j}+c_jD+\pi D(1-F)(1-\beta)\\&-(1-F)pI_e\beta DM_j-cI_cDM_jF+\frac{c_bq_j\beta(1-F)^2}{2\left[(1-\beta)F+\beta\right]}+\frac{(h_o+cI_c)q_jF^2}{2\left[(1-\beta)F+\beta\right]}\end{aligned}. \quad \text{(F1)}$$

Taking the first and second derivatives of Eq. (F1) with respect to F, we have

$$\begin{aligned}\frac{dATC_{11}^{(j)}(F)}{dF}=&\frac{\left[2AD+(cI_c-pI_e)D^2M_j^2\right](1-\beta)}{2q_j}-\pi D(1-\beta)+pI_e\beta DM_j-cI_cDM_j-\\&\frac{c_bq_j\beta\left[(1-F)(1+F)(1-\beta)+2(1-F)\beta\right]}{2\left[(1-\beta)F+\beta\right]^2}+\frac{(h_o+cI_c)q_j\left[(1-\beta)F^2+2F\beta\right]}{2\left[(1-\beta)F+\beta\right]^2};\end{aligned} \quad \text{(F2)}$$

$$\frac{d^2ATC_{11}^{(j)}(F)}{dF^2}=c_bq_j\beta\left\{\frac{1}{(1-\beta)F+\beta}+\frac{(1-\beta)\left[(1-F)(1+F)(1-\beta)+2(1-F)\beta\right]}{\left[(1-\beta)F+\beta\right]^3}\right\}+\frac{(h_o+cI_c)q_j\beta^2}{\left[(1-\beta)F+\beta\right]^3}>0. \quad \text{(F3)}$$

From Eq. (F3), we know that $ATC_{11}^{(j)}(F)$ is convex. Setting $dATC_{11}^{(j)}(F)/dF=0$ yields

$$\begin{aligned}&\frac{\left[2AD+(cI_c-pI_e)D^2M_j^2\right](1-\beta)}{2q_j}-\pi D(1-\beta)+pI_e\beta DM_j-cI_cDM_j+\\&\frac{c_bq_j\beta\left[(1-F)(1+F)(1-\beta)+2(1-F)\beta\right]}{2\left[(1-\beta)F+\beta\right]^2}+\frac{(h_o+cI_c)q_j\left[(1-\beta)F^2+2F\beta\right]}{2\left[(1-\beta)F+\beta\right]^2}=0\end{aligned}. \quad \text{(F4)}$$

After some transformation, the Eq. (F4) can be simplified to

$$\mu_{111}F^2+\mu_{112}F+\mu_{113}=0, \quad \text{(F5)}$$

where

$$\begin{cases}\mu_{111}=2\omega_{111}(1-\beta)^2+c_b\beta(1-\beta)q_j+(h_o+cI_c)(1-\beta)q_j\\\mu_{112}=4\omega_{111}\beta(1-\beta)+2c_b\beta^2q_j+2(h_o+cI_c)\beta q_j\\\mu_{113}=2\omega_{111}\beta^2-c_b(1+\beta)\beta q_j\\\omega_{111}=\frac{\left[2AD+(cI_c-pI_e)D^2M_j^2\right](1-\beta)}{2q_j}-\pi D(1-\beta)+pI_e\beta DM_j-cI_cDM_j\end{cases}. \quad \text{(F5.1)}$$

For the quadratic Eq. (5), if the discriminant ($\Delta=\mu_{112}^2-4\mu_{111}\mu_{113}$) of the Eq. (F5) is negative (i.e., the Eq. (F5) has no roots), which means $\frac{dATC_{11}^{(j)}(F)}{dF}$ is always negative or positive. Therefore, we can conclude that $ATC_{11}^{(j)}(F)$ is strictly decreasing or strictly increasing in $[0,1]$. As a result, $ATC_{11}^{(j)}(F)$ reaches the global minimum at $F_{11}''=1$ or $F_{11}''=0$.

Instead, if the discriminant (Δ) of the Eq. (F5) is no-negative (i.e., the Eq. (F5) has roots). Also from Eq. (F3), we know that $ATC_{11}^{(j)}(F)$ is strictly convex in $[0,1]$, which means the quadratic equation (F5) has zero or one positive root. And because the sign of ω_{111} is indeterminate, thus we may define F_{11}'' is

$$F_{11}''=\max\left\{\frac{-\mu_{112}+\sqrt{\mu_{112}^2-4\mu_{111}\mu_{113}}}{2\mu_{111}},\frac{-\mu_{112}-\sqrt{\mu_{112}^2-4\mu_{111}\mu_{113}}}{2\mu_{111}}\right\}. \tag{F6}$$

Further, if F_{11}'' is feasible, then we obtain the retailer's replenishment cycle

$$T_{11}''=\frac{q_j}{D\left[(1-\beta)K_{11}''+\beta\right]}. \tag{F7}$$

If F_{11}'' is not feasible, we may set $F_{11}''=1$ or $F_{11}''=0$.

In summary, for the solution of F_{11}'' and T_{11}'' derived for Case 1-1, we also need to check whether the constraint $M_j\le F_{11}''T_{11}''\le T_w$ is satisfied. If the constraint is valid, the optimal solution is obtained. Otherwise, the optimal solution does not exist.

Following the same steps used in Case 1-1, we can analyze the rest of the cases, separately.

$$F_{12}''=\max\left\{\frac{-\mu_{122}+\sqrt{\mu_{122}^2-4\mu_{121}\mu_{123}}}{2\mu_{121}},\frac{-\mu_{122}-\sqrt{\mu_{122}^2-4\mu_{121}\mu_{123}}}{2\mu_{121}}\right\}, \tag{F8}$$

where

$$\begin{cases}\mu_{121}=2\omega_{121}(1-\beta)^2+c_b\beta(1-\beta)q_j+(h_o+pI_e)(1-\beta)q_j\\ \mu_{122}=4\omega_{121}\beta(1-\beta)+2c_b\beta^2q_j+2(h_o+pI_e)\beta q_j\\ \mu_{123}=2\omega_{121}\beta^2-c_b(1+\beta)\beta q_j\\ \omega_{121}=\dfrac{AD(1-\beta)}{2q_j}-\pi D(1-\beta)-pI_e(1-\beta)DM_j\end{cases}; \tag{F8.1}$$

$$F_{21}''=\max\left\{\frac{-\mu_{212}+\sqrt{\mu_{212}^2-4\mu_{211}\mu_{213}}}{2\mu_{211}},\frac{-\mu_{212}-\sqrt{\mu_{212}^2-4\mu_{211}\mu_{213}}}{2\mu_{211}}\right\}, \tag{F9}$$

where

$$\begin{cases}\mu_{211}=2\omega_{211}(1-\beta)^2+c_b\beta(1-\beta)q_j+(h_r+cI_c)(1-\beta)q_j\\ \mu_{212}=4\omega_{211}\beta(1-\beta)+2c_b\beta^2q_j+2(h_r+cI_c)\beta q_j\\ \mu_{213}=2\omega_{211}\beta^2-c_b(1+\beta)\beta q_j\\ \omega_{211}=\dfrac{AD(1-\beta)}{q_j}-\pi D(1-\beta)+\dfrac{(h_r-h_o)W^2(1-\beta)}{2q_j}-(h_r-h_o)W-(cI_c-pI_e\beta)DM_j;\\ \quad+\dfrac{(cI_c-pI_e)D^2M_j^2(1-\beta)}{2q_j}\end{cases} \tag{F9.1}$$

$$F_{22}'' = \max\left\{\frac{-\mu_{222}+\sqrt{\mu_{222}^2-4\mu_{221}\mu_{223}}}{2\mu_{221}}, \frac{-\mu_{222}-\sqrt{\mu_{222}^2-4\mu_{221}\mu_{223}}}{2\mu_{221}}\right\}, \quad \text{(F10)}$$

where

$$\begin{cases}\mu_{221} = 2\omega_{221}(1-\beta)^2 + c_b\beta(1-\beta)q_j + (h_r + pI_e)(1-\beta)q_j \\ \mu_{222} = 4\omega_{221}\beta(1-\beta) + 2c_b\beta^2 q_j + 2(h_r + pI_e)\beta q_j \\ \mu_{223} = 2\omega_{221}\beta^2 - c_b(1+\beta)\beta q_j \\ \omega_{221} = \dfrac{2AD + (h_r - h_o)W^2}{2q_j}(1-\beta) - \pi D(1-\beta) - (h_r - h_o)W - (1-\beta)pI_e DM_j \end{cases}. \quad \text{(F10.1)}$$

***Supplement* G. The specific steps of sub-procedures A, B, C, and D.**

Sub-procedure A: Determine $\left(F_{11}^{**}, T_{11}^{**}\right)$ and $ATP_{11}^{(j)}\left(F_{11}^{**}, T_{11}^{**}\right)$

A1. Calculate $\psi_{11i}\,(i = 1,2,\cdots,6)$ from Eqs. (12)–(17). If $\psi_{115} > 0$, execute step A2; if not, execute step A6.

A2. Calculate β_{11} from Eq. (26), if $\beta \le \beta_{11}$, execute step A4; else if $\beta > \beta_{11}$, calculate T_{11} from Eq. (27). If T_{11} is feasible, execute step A3; if not, execute step A4.

A3. Compute F_{11} from Eq. (28), if $F_{11} \le 1$, execute step A5; if not, execute step A4.

A4. Set $F_{11} = 1$, determine $T_{11}^{\#}$ from Eq. (D4) in Supplement D. If $T_{11}^{\#} > T_w$, set $\left(F_{11}^*, T_{11}^*\right) = (1, T_w)$ and execute step A7; else if $T_{11}^{\#} < M_j$, set $\left(F_{11}^*, T_{11}^*\right) = (1, M_j)$ and execute step A7; otherwise, set $\left(F_{11}^*, T_{11}^*\right) = \left(1, T_{11}^{\#}\right)$ and execute step A7.

A5. If $M_j \le F_{11}T_{11} \le T_w$, set $\left(F_{11}^*, T_{11}^*\right) = \left(F_{11}, T_{11}\right)$ and execute step A7; if not, execute step A6.

A6. If $F_{11}T_{11} > T_w$, obtain $\left(F_{11}^*, T_{11}^*\right) = \left(F_{11}', T_{11}'\right)$ by employing Table 2. Then if T_{11}^* and F_{11}^* are feasible, execute step A7; if not, execute step A4. On the other hand, if $F_{11}T_{11} < M_j$, obtain $\left(F_{11}^*, T_{11}^*\right) = \left(F_{11}', T_{11}'\right)$ using Table 2. Now, if T_{11}^* and F_{11}^* are feasible, execute step A7; if not, execute step A4.

A7. Calculate order quantity $Q_j = DT_{11}^*\left[\left(1-F_{11}^*\right)\beta + F_{11}^*\right]$ from Eq. (29), and execute step A8.

A8. Determine the relationship between Q_j and $\left[q_j, q_{j+1}\right)$ using the following sub-steps.

A8.1. If $q_j \le Q_j < q_{j+1}$, set $\left(F_{11}^{**}, T_{11}^{**}\right) = \left(F_{11}^*, T_{11}^*\right)$. Calculate the retailer's annual profit $ATP_{11}^{(j)}\left(F_{11}^{**}, T_{11}^{**}\right)$ using Eq. (7) and execute step A9.

A8.2. If $Q_j \ge q_{j+1}$, then T_{11}^* and F_{11}^* are not feasible solutions, set $ATP_{11}^{(j)}(F,T) = -\inf$.

A8.3. If $Q_j < q_j$, then T_{11}^* and F_{11}^* are not feasible solutions. However, $ATP_{11}^{(j)}(F,T)$ at point $T = \dfrac{q_j}{D\left[(1-F)\beta + F\right]}$ has a maximum value. Thus, calculate F_{11}'' from Eq. (F6) in Supplement F. If F_{11}'' is feasible, execute step A8.3.1; if not, execute step A8.3.2.

A8.3.1. If $M_j \le F_{11}'' T_{11}'' \le T_w$, set $\left(F_{11}^{**}, T_{11}^{**}\right) = \left(F_{11}'', T_{11}''\right)$, and calculate the retailer's annual profit $ATP_{11}^{(j)}\left(F_{11}^{**}, T_{11}^{**}\right)$ using Eq. (7), execute step A9. Otherwise, T_{11}'' and F_{11}'' are not feasible solutions, set $ATP_{11}^{(j)}(F,T) = -\inf$, execute step A9.

A8.3.2. Let $F_{11}'' = 1$ and $T_{11}'' = q_j / D$. If $M_j \le F_{11}'' T_{11}'' \le T_w$, set $\left(F_{11}^{**}, T_{11}^{**}\right) = \left(1, q_j / D\right)$, and calculate the retailer's annual profit $ATP_{11}^{(j)}\left(F_{11}^{**}, T_{11}^{**}\right)$ using Eq. (7), execute step A9. Otherwise, T_{11}'' and F_{11}'' are not feasible solutions, set $ATP_{11}^{(j)}(F,T) = -\inf$, execute step A9.

A9. If $ATP_{11}^{(j)}\left(F_{11}^{**}, T_{11}^{**}\right) \ge -\pi D$, the optimal solutions F_{11}^{**} and T_{11}^{**} are found and stop. Otherwise, execute step A10.

A10. Set $\left(F_{11}^{**}, T_{11}^{**}\right) = (0, \infty)$, $ATP_{11}^{(j)}\left(F_{11}^{**}, T_{11}^{**}\right) = -\pi D$.

Sub-procedure B: Determine $\left(F_{12}^{**}, T_{12}^{**}\right)$ and $ATP_{12}^{(j)}\left(F_{12}^{**}, T_{12}^{**}\right)$

B1. Calculate $\psi_{12i}\ (i = 1, 2, \cdots, 6)$ from Eqs (31)–(36), execute step B2.

B2. Calculate β_{12} from Eq. (38), if $\beta \le \beta_{12}$, execute step B4, else if $\beta > \beta_{12}$, calculate T_{12} from Eq. (39). If T_{12} is feasible, execute step B3, if not, execute step B4.

B3. Calculate F_{12} from Eq. (40), if $F_{12} \le 1$, execute step B5, if not, execute step B4.

B4. Set $F_{12} = 1$, determine $T_{12}^{\#}$ from Eq. (D5) in Supplement D, if $T_{12}^{\#} > \min\{T_w, M_j\}$, set $\left(F_{12}^{*}, T_{12}^{*}\right) = \left(1, \min\{T_w, M_j\}\right)$ and execute step B7; otherwise, set $\left(F_{12}^{*}, T_{12}^{*}\right) = \left(1, T_{12}^{\#}\right)$ and execute step B7.

B5. If $F_{12} T_{12} \le \min\{T_w, M_j\}$, set $\left(F_{12}^{*}, T_{12}^{*}\right) = \left(F_{12}, T_{12}\right)$ and execute step B7; if not, execute step B6.

B6. If $F_{12} T_{12} > \min\{T_w, M_j\}$, obtain $\left(F_{12}^{*}, T_{12}^{*}\right) = \left(F_{12}', T_{12}'\right)$ by employing Table 2. Then if T_{12}^{*} and F_{12}^{*} are feasible, execute step B7; if not, execute step B4.

B7. Calculate order quantity $Q_j = D T_{12}^{*}\left[\left(1 - F_{12}^{*}\right)\beta + F_{12}^{*}\right]$, and execute step B8.

B8. Determine the relationship between Q_j and $\left[q_j, q_{j+1}\right)$ using the following sub-steps.

B8.1. If $q_j \le Q_j < q_{j+1}$, set $\left(F_{12}^{**}, T_{12}^{**}\right) = \left(F_{12}^{*}, T_{12}^{*}\right)$. Calculate the retailer's annual profit $ATP_{12}^{(j)}\left(F_{12}^{**}, T_{12}^{**}\right)$ using Eq. (8) and execute step B9.

B8.2. If $Q_j \ge q_{j+1}$, then T_{12}^{*} and F_{12}^{*} are not feasible solutions, set $ATP_{12}^{(j)}\left(F_{12}^{**}, T_{12}^{**}\right) = -\inf$.

B8.3. If $Q_j < q_j$, then T_{12}^{*} and F_{12}^{*} are not feasible solutions. However, $ATP_{12}^{(j)}(F,T)$ at point $T = \dfrac{q_j}{D\left[(1-F)\beta + F\right]}$ has a maximum value. Thus, calculate F_{12}'' from Eq. (F8) in Supplement F. If F_{12}'' is feasible, execute step B8.3.1; if not, execute step B8.3.2.

B8.3.1. If $F''_{12}T''_{12} \le \min\{T_w, M_j\}$, set $(F^{**}_{12}, T^{**}_{12}) = (F''_{12}, T''_{12})$, and calculate the retailer's annual profit $ATP^{(j)}_{12}(F^{**}_{12}, T^{**}_{12})$ using Eq. (8), execute step B9. Otherwise, T''_{12} and F''_{12} are not feasible solutions, set $ATP^{(j)}_{12}(F^{**}_{12}, T^{**}_{12}) = -\inf$, execute step B9.

B8.3.2. Let $F''_{12} = 1$ and $T''_{12} = q_j / D$. If $F''_{12}T''_{12} \le \min\{T_w, M_j\}$, set $(F^{**}_{12}, T^{**}_{12}) = (1, q_j / D)$, and calculate the retailer's annual profit $ATP^{(j)}_{12}(F^{**}_{12}, T^{**}_{12})$ using Eq. (8), execute step B9. Otherwise, T''_{12} and F''_{12} are not feasible solutions, set $ATP^{(j)}_{12}(F^{**}_{12}, T^{**}_{12}) = -\inf$, execute step B9.

B9. If $ATP^{(j)}_{12}(F^{**}_{12}, T^{**}_{12}) \ge -\pi D$, the optimal solutions F^{**}_{12} and T^{**}_{12} are found and stop. Otherwise, execute step B10.

B10. Set $(F^{**}_{12}, T^{**}_{12}) = (0, \infty)$, $ATP^{(j)}_{12}(F^{**}_{12}, T^{**}_{12}) = -\pi D$.

Sub-procedure C: Determine $(F^{**}_{21}, T^{**}_{21})$ and $ATP^{(j)}_{21}(F^{**}_{21}, T^{**}_{21})$

C1. Calculate $\psi_{21i}(i = 1, 2, \cdots, 6)$ from Eqs (42)–(47). If $\psi_{215} > 0$, execute step C2; if not, execute step C6.

C2. Calculate β_{21} from Eq. (49), if $\beta \le \beta_{21}$, execute step C4; else if $\beta > \beta_{21}$, calculate T_{21} from Eq. (50). If T_{21} is feasible, execute step C3; if not, execute step C4.

C3. Compute F_{21} from Eq. (51), if $F_{21} \le 1$, execute step C5; if not, execute step C4.

C4. Set $F_{21} = 1$, determine $T^{\#}_{21}$ from Eq. (D6) in Supplement D. If $T^{\#}_{21} < \max\{T_w, M_j\}$, set $(F^{*}_{21}, T^{*}_{21}) = (1, \max\{T_w, M_j\})$ and execute step C7. Otherwise, set $(F^{*}_{21}, T^{*}_{21}) = (1, T^{\#}_{21})$ and execute step C7.

C5. If $\max\{T_w, M_j\} \le F_{21}T_{21}$, set $(F^{*}_{21}, T^{*}_{21}) = (F_{21}, T_{21})$ and execute step C7; if not, execute step C6.

C6. If $F_{21}T_{21} < \max\{T_w, M_j\}$, obtain $(F^{*}_{21}, T^{*}_{21}) = (F'_{21}, T'_{21})$ by employing Table 2 Then if T^{*}_{21} and F^{*}_{21} are feasible, execute step C7; if not, execute step C4.

C7. Calculate order quantity $Q_j = DT^{*}_{21}\left[(1 - F^{*}_{21})\beta + F^{*}_{21}\right]$, and execute step C8.

C8. Determine the relationship between Q_j and $[q_j, q_{j+1})$ using the following sub-steps.

C8.1. If $q_j \le Q_j < q_{j+1}$, set $(F^{**}_{21}, T^{**}_{21}) = (F^{*}_{21}, T^{*}_{21})$. Calculate the retailer's annual profit $ATP^{(j)}_{21}(F^{**}_{21}, T^{**}_{21})$ using Eq. (9) and execute step (C9).

C8.2. If $Q_j \ge q_{j+1}$, then T^{*}_{21} and F^{*}_{21} are not feasible solutions, set $ATP^{(j)}_{21}(F^{**}_{21}, T^{**}_{21}) = -\inf$.

C8.3. If $Q_j < q_j$, then T^{*}_{21} and F^{*}_{21} are not feasible solutions. However, $ATP^{(j)}_{21}(F, T)$ at point $T = \dfrac{q_j}{D\left[(1 - F)\beta + F\right]}$ has a maximum value. Thus, calculate F''_{21} from Eq. (F9) in Supplement F. If F''_{21} is feasible, execute step C8.3.1; if not, execute step C8.3.2.

C8.3.1. If $\max\{T_w, M_j\} \le F''_{21}T''_{21}$, set $\left(F_{21}^{**}, T_{21}^{**}\right) = \left(F''_{21}, T''_{21}\right)$, and calculate the retailer's annual profit $ATP_{21}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right)$ using Eq. (9), execute step C9. Otherwise, T''_{21} and F''_{21} are not feasible solutions, set $ATP_{21}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right) = -\inf$, execute step C9.

C8.3.2. Let $F''_{21} = 1$ and $T''_{21} = q_j/D$. If $\max\{T_w, M_j\} \le F''_{21}T''_{21}$, set $\left(F_{21}^{**}, T_{21}^{**}\right) = \left(1, q_j/D\right)$, and calculate the retailer's annual profit $ATP_{21}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right)$ using Eq. (9), execute step C9. Otherwise, T''_{21} and F''_{21} are not feasible solutions, set $ATP_{21}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right) = -\inf$, execute step C9.

C9. If $ATP_{21}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right) \ge -\pi D$, the optimal solutions F_{21}^{**} and T_{21}^{**} are found and stop. Otherwise, execute step C10.

C10. Set $\left(F_{21}^{**}, T_{21}^{**}\right) = (0, \infty)$, $ATP_{21}^{(j)}\left(F_{21}^{**}, T_{21}^{**}\right) = -\pi D$.

Sub-procedure D: Determine $\left(F_{22}^{**}, T_{22}^{**}\right)$ and $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right)$

D1. Calculate $\psi_{22i}\ (i = 1, 2, \cdots, 6)$ from Eqs (53)–(58) and execute step D2.

D2. Calculate β_{22} from Eq. (60), if $\beta \le \beta_{22}$, execute D4; else if $\beta > \beta_{22}$, calculate T_{22} from Eq. (61). If T_{22} is feasible, execute step D3; if not, execute step D4.

D3. Compute F_{22} from Eq. (62), if $F_{22} \le 1$, execute step D5; if not, execute step D4.

D4. Set $F_{22} = 1$, determine $T_{22}^{\#}$ from Eq. (D7) in Supplement D. If $T_{22}^{\#} < T_w$, set $\left(F_{22}^{*}, T_{22}^{*}\right) = \left(1, T_w\right)$ and execute step D7; else if $T_{22}^{\#} > M_j$, set $\left(F_{22}^{*}, T_{22}^{*}\right) = \left(1, M_j\right)$ and execute step D7; otherwise, set $\left(F_{22}^{*}, T_{22}^{*}\right) = \left(1, T_{22}^{\#}\right)$ and execute step D7.

D5. If $T_w \le F_{22}T_{22} \le M_j$, set $\left(F_{22}^{*}, T_{22}^{*}\right) = \left(F_{22}, T_{22}\right)$ and execute step D7; if not, execute step D6.

D6. If $F_{22}T_{22} < T_w$, obtain $\left(F_{22}^{*}, T_{22}^{*}\right) = \left(F'_{22}, T'_{22}\right)$ by employing Table 2. Then if T_{22}^{*} and F_{22}^{*} are feasible, execute step D7; if not, execute step D4. On the other hand, if $F_{22}T_{22} > M_j$, obtain $\left(F_{22}^{*}, T_{22}^{*}\right) = \left(F'_{22}, T'_{22}\right)$ using Table 2. Now, if T_{22}^{*} and F_{22}^{*} are feasible, execute step D7; if not, execute step D4.

D7. Calculate order quantity $Q_j = DT_{22}^{*}\left[\left(1 - F_{22}^{*}\right)\beta + F_{22}^{*}\right]$, and execute step D8.

D8. Determine the relationship between Q_j and $\left[q_j, q_{j+1}\right)$ using the following sub-steps.

D8.1. If $q_j \le Q_j < q_{j+1}$, set $\left(F_{22}^{**}, T_{22}^{**}\right) = \left(F_{22}^{*}, T_{22}^{*}\right)$. Calculate the retailer's annual profit $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right)$ using Eq. (10) and execute step D9.

D8.2. If $Q_j \ge q_{j+1}$, then T_{22}^{*} and F_{22}^{*} are not feasible solutions, set $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right) = -\inf$.

D8.3. If $Q_j < q_j$, then T_{22}^{*} and F_{22}^{*} are not feasible solutions. However, $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right)$

at point $T = \dfrac{q_j}{D\left[(1-F)\beta + F\right]}$ has a maximum value. Thus, calculate F_{22}'' from Eq. (F10) in Supplement F. If F_{22}'' is feasible, execute step D8.3.1; if not, execute step D8.3.2.

D8.3.1. If $T_w \le F_{22}''T_{22}'' \le M_j$, set $\left(F_{22}^{**}, T_{22}^{**}\right) = \left(F_{22}'', T_{22}''\right)$, and calculate the retailer's annual profit $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right)$ using Eq. (10), execute step D9. Otherwise, T_{22}'' and F_{22}'' are not feasible solutions, set $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right) = -\inf$, execute step D9.

D8.3.2. Let $F_{22}'' = 1$ and $T_{22}'' = q_j/D$. If $T_w \le F_{22}''T_{22}'' \le M_j$, set $\left(F_{22}^{**}, T_{22}^{**}\right) = \left(1, q_j/D\right)$, and calculate the retailer's annual profit $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right)$ using Eq. (10), execute step D9. Otherwise, T_{22}'' and F_{22}'' are not feasible solutions, set $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right) = -\inf$, execute step D9.

D9. If $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right) \ge -\pi D$, the optimal solutions F_{22}^{**} and T_{22}^{**} are found and stop. Otherwise, execute step D10

D10. Set $\left(F_{22}^{**}, T_{22}^{**}\right) = (0, \infty)$, $ATP_{22}^{(j)}\left(F_{22}^{**}, T_{22}^{**}\right) = -\pi D$.