METHODS FOR PROBABILISTIC DECISION MAKING WITH LINGUISTIC INFORMATION

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Abstract. With respect to decision making problems by using probabilities, immediate probabilities and information that can be represented with linguistic labels, some new decision analysis are proposed. Firstly, we shall develop three new aggregation operators: generalized probabilistic 2-tuple weighted average (GP-2TWA) operator, generalized probabilistic 2-tuple ordered weighted average (GP-2TOWA) operator and generalized immediate probabilistic 2-tuple ordered weighted average (GIP-2TOWA) operator. These operators use the weighted average (WA) operator, the ordered weighted average (OWA) operator, linguistic information, probabilistic information and immediate probabilistic information. They are quite useful because they can assess the uncertain information within the problem by using both linguistic labels and the probabilistic information that considers the attitudinal character of the decision maker. In these approaches, alternative appraisal values are calculated by the aggregation of 2-tuple linguistic information. Thus, the ranking of alternative or selection of the most desirable alternative(s) is obtained by the comparison of 2-tuple linguistic information. Finally, we give an illustrative example about selection of strategies to verify the developed approach and to demonstrate its feasibility and practicality.

Keywords: decision making, linguistic information, probabilistic, generalized probabilistic 2-tuple weighted average (GP-2TWA) operator, generalized probabilistic 2-tuple ordered weighted average (GP-2TOWA) operator, generalized immediate probabilistic 2-tuple ordered weighted average (GIP-2TOWA) operator.

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Introduction

A multiple attribute decision making problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and

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qualitative. In order to choose a desirable solution, decision makers often provide his/her preference information which takes the form of numerical values, such as exact values, interval number values and fuzzy numbers (Liu 2009; Zhang, Liu 2010; Han, Liu 2011; Kaya, Kahraman 2011; Yan *et al*. 2011; Wei 2010a, b, 2012). However, under many conditions, numerical values are inadequate or insufficient to model real-life decision problems. Indeed, human judgments including preference information may be stated in linguistic terms. Thus, multiple attribute decision making problems under linguistic environment is an interesting research topic having received more and more attention from researchers during the last several years. Several methods have been proposed for dealing with linguistic information. These methods are mainly as follows:

(1) The approximative computational model based on the Extension Principle (Degani, Bortolan 1988);

(2) The ordinal linguistic computational model (Delgado *et al*. 1993);

(3) The 2-tuple linguistic computational model (Herrera, Herrera-Viedma 2000a, b; Herrera *et al*. 2005; Herrera, Martínez 2001; Wei 2010a, b, 2011a, c; Wei, Zhao 2012).

The first models transform linguistic assessment information into fuzzy numbers and uses fuzzy arithmetic to make computations over these fuzzy numbers. The use of fuzzy arithmetic increases the vagueness. The results obtained by the fuzzy arithmetic are fuzzy numbers that usually do not match any linguistic term in the initial term set. The second models is also called symbolic model which makes direct computations on labels using the ordinal structure of the linguistic term sets. But symbolic method easily results in a loss of information caused by the use of the round operator. The third models use the 2-tuple linguistic representation and computational model to make linguistic computations. Herrera and Martínez (1991) show 2-tuple linguistic information processing manner can effectively avoid the loss and distortion of information. It has a distinct advantage over other linguistic processing methods in accuracy and reliability. Herrera, Herrera-Viedma (2000a) developed 2-tuple arithmetic average (TAA) operator, 2-tuple weighted average (TWA) operator, 2-tuple ordered weighted average (TOWA) operator and extended 2-tuple weighted average (ET-WA) operator. Herrera *et al*. (2005) presented a group decision making process for managing non-homogeneous information. The non-homogeneous information can be represented as values belonging to domains with different nature as linguistic, numerical and interval valued or can be values assessed in label sets with different granularity, multi-granular linguistic information. Herrera-Viedma *et al*. (2005) presented a model of consensus support system to assist the experts in all phases of the consensus reaching process of group decision-making problems with multi-granular linguistic preference relations. Liao *et al*. (2007) presented a model for selecting an ERP system based on linguistic information processing. Herrera *et al*. (2008) proposed a fuzzy linguistic methodology to deal with unbalanced linguistic term sets. Wang (2009) presented a 2-tuple fuzzy linguistic evaluation model for selecting appropriate agile manufacturing system in relation to MC production. Tai and Chen (2009) developed a new evaluation model for intellectual capital based on computing with linguistic variable. Fan *et al*. (2009) Evaluated knowledge management capability of organizations by using a fuzzy linguistic method. Zhang and Chu (2009) developed fuzzy group decision making for multi-format and multi-granularity linguistic judgments in quality function deployment.

Wei (2010a) extended TOPSIS method to 2-tuple linguistic multiple attribute group decision making with incomplete weight information. Wei (2010b) proposed a method for multiple attribute group decision making based on the ET-WG and ET-OWG operators with 2-tuple linguistic information. Fan and Liu (2010) developed a method for group decision making based on multi-granularity uncertain linguistic information. Chang and Wen (2010) developed a novel efficient approach for DFMEA combining 2-tuple and the OWA operator. Ngan (2011) took an alternative view that the result of aggregating fuzzy ratings should be fuzzy itself, and therefore they further developed the 2-tuple fuzzy linguistic methodology so that its output is a fuzzy number describing the aggregation of opinions. they demonstrated the utility of the extended fuzzy linguistic computing methodology by applying it to two data sets: (i) the evaluation of a new product idea in a Taiwanese electronics manufacturing firm and (ii) the evaluation of the investment benefit of a proposed facility site.

Another interesting approach is the use of probabilistic information in the analysis because we are able to introduce objectivity in our studies. In the literature, we find a wide range of models that use probabilistic information such as the immediate probabilities (Yager *et al*. 1995; Merigó 2010) and the Dempster-Shafer theory of evidence (Merigó, Casanovas 2009; Merigó *et al*. 2010). Recently, Merigó (2008, 2009a) has suggested the probabilistic OWA (POWA) operator. It is an aggregation operator that unifies the probability with the OWA operator considering the degree of importance that each concept has in the aggregation. Moreover, he has also suggested the probabilistic weighted average (PWA) (Merigó 2009b). Its main advantage is that it unifies the probability and the weighted average in the same formulation considering how relevant they are in the aggregation.

In this paper, we investigate the probabilistic decision making problems with linguistic information, some new probabilistic decision making methods are developed. The aim of this paper is to develop some new decision making models by using probabilities, immediate probabilities and information that can be represented with linguistic labels. Firstly, we shall develop three new aggregation operators: generalized probabilistic 2-tuple weighted average (GP-2TWA) operator, generalized probabilistic 2-tuple ordered weighted average (GP-2TOWA) operator and generalized immediate probabilistic 2-tuple ordered weighted average (GIP-2TOWA) operator. These operators use the weighted average (WA) operator, the ordered weighted average (OWA) operator, linguistic information and probabilistic information. They are quite useful because they can assess the uncertain information within the problem by using both linguistic labels and the probabilistic information that considers the attitudinal character of the decision maker. In doing so, the remainder of this paper is set out as follows. In the next section, we introduce some basic concepts and operational laws of 2-tuple linguistic variables. In Section 2 we develop some generalized probabilistic weighted average operator with linguistic assessment information, some generalized probabilistic ordered weighted average operator with linguistic assessment information and some generalized immediate probabilistic ordered weighted average operator with linguistic assessment information. In Section 3, we give an illustrative example about selection of strategies to verify the developed approach and to demonstrate its feasibility and practicality. In the final section we conclude the paper and give some remarks.

1. Preliminaries

Let $S = \{s_i | i = 1,2,\cdots,t\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics (Herrera, Herrera-Viedma 2000a, b; Herrera *et al.* 2005; Herrera, Martínez 2001):

(1) The set is ordered: $s_i > s_j$, if $i > j$; (2) Max operator: $\max(s_i, s_j) = s_i$, if ..; (3) Min operator: $\min(s_i, s_j) = s_i$, if $s_i \le s_j$. For example, S can be defined as:

$$S = \{s_1 = extremely\ poor, s_2 = very\ poor, s_3 = poor, s_4 = medium,$$
$$s_5 = good, s_6 = very\ good, s_7 = extremely\ good\}.$$

Herrera, Herrera-Viedma (2000a, b) developed the 2-tuple fuzzy linguistic representation model based on the concept of symbolic translation. It is used for representing the linguistic assessment information by means of a 2-tuple (s_i, α_i), where s_i is a linguistic label from pre-defined linguistic term set S and α_i is the value of symbolic translation, and $\alpha_i \in [-0.5, 0.5)$.

Definition 1. Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e. the result of a symbolic aggregation operation, $\beta \in [1, t]$, and t the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be 2 values, such that $i \in [1, t]$ and $\alpha \in [-0.5, 0.5)$; then α is called a symbolic translation of the aggregation (Herrera, Herrera-Viedma 2000a, b).

Definition 2. Let $S = \{s_1, s_2, \cdots, s_t\}$ be a linguistic term set and $\beta \in [1, t]$ be a value representing the result of a symbolic aggregation operation; then 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [1, t] \rightarrow S \times [-0.5, 0.5); \tag{1}$$

$$\Delta(\beta) = \begin{cases} s_i \cdot i = round(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5) \end{cases}, \tag{2}$$

where: round(.) is the usual round operation; s_i has the closest index label to β and α is the value of the symbolic translation (Herrera, Herrera-Viedma 2000a, b).

Definition 3. Let $S = \{s_1, s_2, \cdots, s_t\}$ be a linguistic term set and (s_i, α_i) be a 2-tuple; a function Δ^{-1} can be defined, such that, from a 2-tuple (s_i, α_i) it return its equivalent numerical value $\beta \in [1, t] \subset R$, which is obtained with the following function (Herrera, Herrera-Viedma 2000a, b):

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [1, t]; \tag{3}$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \tag{4}$$

Definition 4. The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order. Let (s_k, a_k) and (s_l, a_l) be two 2-tuples, with each one representing a linguistic assessment (Herrera, Herrera-Viedma 2000a, b):

(1) If $k < l$ then (s_k, a_k) is smaller than (s_l, a_l);

(2) If $k = l$ then, 1) if $a_k = a_l$, then (s_k, a_k), (s_l, a_l) represents the same information; 2) if $a_k < a_l$ then (s_k, a_k) is smaller than (s_l, a_l); 3) if $a_k > a_l$ then (s_k, a_k) is bigger than (s_l, a_l).

2. Some generalized probabilistic weighted average operator with linguistic information

Merigó (2008, 2009a) developed the probabilistic weighted average (PWA) operator which is an aggregation operator that unifies the probability and the weighted average in the same formulation considering the degree of importance that each concept has in the aggregation.

Definition 5. An PWA operator of dimension n is a mapping PWA: $R^n \rightarrow R$, such that:

$$\mathrm{PWA}(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} v_j a_j,$$ (5)

where: $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of $a_j (j = 1, 2, \cdots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$, and a probabilistic weight $p_j > 0$, $\sum_{j=1}^{n} p_j = 1$, $v_j = \beta p_j + (1 - \beta)\omega_j$ with $\beta \in [0,1]$ and v_j is the weight that unifies probabilities and WAs in the same formulation, then PWA is called the probabilistic weighted average (PWA) operator. The PWA operator is monotonic, commutative, bounded and idempotent.

Merigó (2008) proposed the generalized probabilistic weighted average (GPWA) operator which is an aggregation operator that unifies the probability and the weighted average in the same formulation considering the degree of importance that each concept has in the aggregation.

Definition 6. A GPWA operator of dimension n is a mapping GPWA: $R^n \rightarrow R$, such that:

$$\mathrm{GPWA}(a_1, a_2, \cdots, a_n) = \left(\sum_{j=1}^{n} v_j a_j^{\lambda} \right)^{1/\lambda},$$ (6)

where: $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of $a_j (j = 1, 2, \cdots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$, and a probabilistic weight $p_j > 0$, $\sum_{j=1}^{n} p_j = 1$, $v_j = \beta p_j + (1 - \beta)\omega_j$ with $\beta \in [0,1]$ and v_j is the weight that unifies probabilities and WAs in the same formulation and λ is a parameter such that $\lambda \in (-\infty, +\infty)$.

It's obvious that GPWA operator include a wide range of aggregation operators such as the PWA, the probabilistic weighted geometric (PWG), the probabilistic weighted harmonic average (PHWA), the probabilistic weighted quadratic average (PQWA), and a lot of other cases.

2.1. Generalized probabilistic weighted average operator with linguistic information

In the following, we extend the GPWA operator to linguistic environment and develop the generalized probabilistic 2-tuple weighted average (GP-2TWA) operator. The GP-2TWA is an aggregation operator that unifies the probability and the weighted average in the same formulation considering the degree of importance that each concept has in the aggregation and using uncertain information represented with linguistic values.

Definition 7. Let $\left\{\left(r_1,a_1\right),\left(r_2,a_2\right),\ldots,\left(r_n,a_n\right)\right\}$ be a set of 2-tuple, An GP-2TWA operator of dimension n is a mapping GP-2TWA: $S^n \to S$, furthermore,

$$\text{GP-2TWA}\left(\left(r_1,a_1\right),\left(r_2,a_2\right),\ldots,\left(r_n,a_n\right)\right)=\Delta\left(\left(\sum_{j=1}^{n}v_j\Delta^{-1}\left(r_j,a_j\right)^{\lambda}\right)^{1/\lambda}\right), \quad (7)$$

where: $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)^T$ be the weight vector of $\left(r_j,a_j\right)\left(j=1,2,\cdots,n\right)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, and a probabilistic weight $p_j>0$, $\sum_{j=1}^{n}p_j=1$, $v_j=\beta p_j+\left(1-\beta\right)\omega_j$ with $\beta\in\left[0,1\right]$ and v_j is the weight that unifies probabilities and WAs in the same formulation and λ is a parameter such that $\lambda\in\left(-\infty,+\infty\right)$.

Especially, if $\beta=0$, then, the GP-2TWA operator reduces to the G-2TWA operator. And if $\beta=1$, it becomes the generalized probabilistic 2-tuple average (GP-2TA). Note that the GP-2TWA is monotonic, bounded and idempotent.

Remark 1. When $\lambda=1$, the GP-2TWA operator become the P-2TWA operator.

$$\text{P-2TWA}\left(\left(r_1,a_1\right),\left(r_2,a_2\right),\ldots,\left(r_n,a_n\right)\right)=\Delta\left(\sum_{j=1}^{n}v_j\Delta^{-1}\left(r_j,a_j\right)\right).$$

Note that if $\beta=0$, for all $\left(r_i,a_i\right)$, we get the 2TWA operator; if $\beta=1$, for all $\left(r_i,a_i\right)$, we get the P-2TWA operator.

Remark 2. When $\lambda=0$, the GP-2TWA operator become the probabilistic 2-tuple weighted geoemtric (P-2TWG) operator.

$$\text{P-2TWG}\left(\left(r_1,a_1\right),\left(r_2,a_2\right),\ldots,\left(r_n,a_n\right)\right)=\Delta\left(\prod_{j=1}^{n}\left(\Delta^{-1}\left(r_j,a_j\right)\right)^{v_j}\right).$$

Note that if $\beta=0$, for all $\left(r_i,a_i\right)$, we get the 2TWG operator; if $\beta=1$, for all $\left(r_i,a_i\right)$, we get the P-2TWG operator.

Remark 3. When $\lambda=-1$, the GP-2TWA operator become the probabilistic 2-tuple weighted harmonic average (P-2TWHA) operator.

$$\text{P-2TWHA}\left(\left(r_1,a_1\right),\left(r_2,a_2\right),\ldots,\left(r_n,a_n\right)\right)=\Delta\left(\left(\sum_{j=1}^{n}v_j\Delta^{-1}\left(r_j,a_j\right)^{-1}\right)^{1/(-1)}\right)=$$

$$\Delta\left(\left(\frac{v_j}{\Delta^{-1}\left(r_j,a_j\right)}\right)^{-1}\right)=\Delta\left(\frac{1}{\displaystyle\sum_{j=1}^{n}\frac{v_j}{\Delta^{-1}\left(r_j,a_j\right)}}\right).$$

Note that if $\beta=0$, for all $\left(r_i,a_i\right)$, we get the 2TWHA operator; if $\beta=1$, for all $\left(r_i,a_i\right)$, we get the P-2TWHA operator.

Remark 4. When $\lambda = 2$, the GP-2TWA operator become the probabilistic 2-tuple weighted quadratic average (P-2TWQA) operator.

$$\text{P-2TWQA}\left(\left(r_1,a_1\right),\left(r_2,a_2\right),\ldots,\left(r_n,a_n\right)\right) = \Delta\left(\left(\sum_{j=1}^{n} v_j \Delta^{-1}\left(r_j,a_j\right)^2\right)^{1/2}\right).$$

Note that if $\beta = 0$, for all $\left(r_i,a_i\right)$, we get the 2TWQA operator; if $\beta = 1$, for all $\left(r_i,a_i\right)$, we get the P-2TWQA operator.

2.2. Generalized probabilistic ordered weighted average operator with linguistic information

Another approach for unifying probabilities and OWAs in the same formulation is the probabilistic OWA (POWA) operator (Merigó 2008, 2009b). Its main advantage is that it is able to include both concepts considering the degree of importance of each case in the problem.

Definition 8. An POWA operator of dimension n is a mapping PWA: $R^n \rightarrow R$ that has an associated weight vector $w = \left(w_1, w_2, \cdots, w_n\right)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$, according to the following equation:

$$\text{POWA}\left(a_1, a_2, \cdots, a_n\right) = \sum_{j=1}^{n} \hat{p}_j a_{\sigma(j)}, \tag{8}$$

where: $\left(\sigma(1), \sigma(2), \cdots, \sigma(n)\right)$ is a permutation of $\left(1, 2, \cdots, n\right)$, such that $a_{\sigma(j-1)} \geq a_{\sigma(j)}$ for all $j = 2, \cdots, n$, with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in \left[0, 1\right]$, $\hat{p}_j = \beta p_j + \left(1-\beta\right)w_j$ with $\beta \in \left[0, 1\right]$ and p_j is the associated probability of $a_{\sigma(j)}$.

We shall propose the generalized probabilistic ordered weighted average (GPOWA) operator which is an aggregation operator that unifies the probability and the weighted average in the same formulation considering the degree of importance that each concept has in the aggregation.

Definition 9. A GPOWA operator of dimension n is a mapping GPOWA: $R^n \rightarrow R$ that has an associated weight vector $w = \left(w_1, w_2, \cdots, w_n\right)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$, according to the following equation:

$$\text{GPOWA}\left(a_1, a_2, \cdots, a_n\right) = \left(\sum_{j=1}^{n} \hat{p}_j a_{\sigma(j)}^{\lambda}\right)^{\lambda}, \tag{9}$$

where: $\left(\sigma(1), \sigma(2), \cdots, \sigma(n)\right)$ is a permutation of $\left(1, 2, \cdots, n\right)$, such that $a_{\sigma(j-1)} \geq a_{\sigma(j)}$ for all $j = 2, \cdots, n$, with $\beta \in [0, 1]$ and $p_i \in \left[0, 1\right]$, $\hat{p}_j = \beta p_j + \left(1-\beta\right)w_j$ with $\beta \in \left[0, 1\right]$ and p_j is the associated probability of $a_{\sigma(j)}$ and λ is a parameter such that $\lambda \in \left(-\infty, +\infty\right)$.

It's obvious that GPOWA operator include a wide range of aggregation operators such as the POWA, the probabilistic ordered weighted geometric (POWG), the probabilistic ordered

weighted harmonic average (POWHA), the probabilistic ordered weighted quadratic average (POWQA), and a lot of other cases.

In the following, we extend the GPOWA operator to linguistic environment and develop the generalized probabilistic 2-tuple weighted average (GP-2TOWA) operator.

Definition 10. Let $\{(r_1,a_1),(r_2,a_2),\dots,(r_n,a_n)\}$ be a set of 2-tuple, An GP-2TOWA operator of dimension n is a mapping GP-2TOWA: $S^n \to S$ that has an associated weight vector $w = (w_1,w_2,\cdots,w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\text{GP-2TOWA}\big((r_1,a_1),(r_2,a_2),\dots,(r_n,a_n)\big) = \Delta\left(\left(\sum_{j=1}^{n} \hat{p}_j \Delta^{-1}\big(r_{\sigma(j)},a_{\sigma(j)}\big)^{\lambda}\right)^{1/\lambda}\right), \quad (10)$$

where: $\big(\sigma(1),\sigma(2),\cdots,\sigma(n)\big)$ is a permutation of $(1,2,\cdots,n)$, such that $(r_{\sigma(j-1)},a_{\sigma(j-1)}) \geq (r_{\sigma(j)},a_{\sigma(j)})$ for all $j = 2,\cdots,n$, with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0,1]$, $\hat{p}_j = \beta p_j + (1-\beta)w_j$ with $\beta \in [0,1]$ and p_j is the associated probability of $(r_{\sigma(j)},a_{\sigma(j)})$ and λ is a parameter such that $\lambda \in (-\infty,+\infty)$.

The GP-2TOWA operator unifies the probability and the OWA operator in the same formulation considering the degree of importance of each concept in the aggregation. It also uses information represented in the form of linguistic values.

Remark 5. When $\lambda = 1$, the GP-2TOWA operator reduces to the P-2TOWA operator.

$$\text{P-2TOWA}\big((r_1,a_1),(r_2,a_2),\dots,(r_n,a_n)\big) = \Delta\left(\sum_{j=1}^{n} \hat{p}_j \Delta^{-1}\big(r_{\sigma(j)},a_{\sigma(j)}\big)\right).$$

Note that if $\beta = 0$, for all (r_i,a_i), we get the 2TOWA operator; if $\beta = 1$, for all (r_i,a_i), we get the P-2TOWA operator.

Remark 6. When $\lambda = 0$, the GP-2TOWA operator reduces to the P-2TOWG operator.

$$\text{P-2TOWG}\big((r_1,a_1),(r_2,a_2),\dots,(r_n,a_n)\big) = \Delta\left(\prod_{j=1}^{n}\big(\Delta^{-1}\big(r_{\sigma(j)},a_{\sigma(j)}\big)\big)^{\hat{p}_j}\right).$$

Note that if $\beta = 0$, for all (r_i,a_i), we get the 2TOWG operator; if $\beta = 1$, for all (r_i,a_i), we get the P-2TOWG operator.

Remark 7. When $\lambda = -1$, the GP-2TOWA operator reduces to the P-2TOWHA operator.

$$\text{P-2TOWHA}\big((r_1,a_1),(r_2,a_2),\dots,(r_n,a_n)\big) = \Delta\left(\left(\sum_{j=1}^{n} \hat{p}_j \Delta^{-1}\big(r_{\sigma(j)},a_{\sigma(j)}\big)^{-1}\right)^{1/(-1)}\right) =$$

$$\Delta\left(\left(\sum_{j=1}^{n} \frac{\hat{p}_j}{\Delta^{-1}\big(r_{\sigma(j)},a_{\sigma(j)}\big)}\right)^{-1}\right) = \Delta\left(\frac{1}{\sum_{j=1}^{n} \dfrac{\hat{p}_j}{\Delta^{-1}\big(r_{\sigma(j)},a_{\sigma(j)}\big)}}\right).$$

Note that if $\beta = 0$, for all (r_i, a_i), we get the 2TOWHA operator; if $\beta = 1$, for all (r_i, a_i), we get the P-2TOWHA operator.

Remark 8. When $\lambda = 2$, the GP-2TOWA operator reduces to the P-2TOWQA operator.

$$\text{P-2TWQA}\big((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\big) = \Delta\left(\left(\sum_{j=1}^{n} \hat{p}_j \Delta^{-1}\left(r_{\sigma(j)}, a_{\sigma(j)}\right)^2\right)^{1/2}\right).$$

Note that if $\beta = 0$, for all (r_i, a_i), we get the 2TOWQA operator; if $\beta = 1$, for all (r_i, a_i), we get the P-2TOWQA operator.

2.3. Generalized immediate probabilistic ordered weighted average operator with linguistic information

The GOWA operator (Yager 2004) is a generalization of the OWA operator (Yager 1998) by using generalized means. It can be defined as follows.

Definition 11. A GOWA operator of dimension n is a mapping GOWA: $R^n \to R$ that has an associated weight vector $w = (w_1, w_2, \cdots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\text{GOWA}(a_1, a_2, \cdots, a_n) = \left(\sum_{j=1}^{n} w_j a_{\sigma(j)}^{\lambda}\right)^{1/\lambda}, \tag{11}$$

where $\big(\sigma(1), \sigma(2), \cdots, \sigma(n)\big)$ is a permutation of $(1, 2, \cdots, n)$, such that $\alpha_{\sigma(j-1)} \geq \alpha_{\sigma(j)}$ for all $j = 2, \cdots, n$.

Merigó (2010) develop the immediate probability (IP) which tries to include the decision maker's attitude in a probabilistic decision-making problem. The main advantage is that it is easy to apply it in almost all the probabilistic problems studied before such as in decision-making problems, actuarial sciences and statistics. Because the probabilistic information is objective but uncertain, we cannot then guarantee that the expected result is the result that will happen in the future. If we are in the situations of uncertainty (risk environments), each decision maker will have different attitudes towards the same problem.

In order to develop the analysis, Merigó (2010) used in the same formulation the weights of the OWA operator and the probabilistic information and proposed the immediate probability OWA (IP-OWA) operator. It can be defined as follows.

Definition 12. An IP-OWA operator of dimension n is a mapping IP-OWA: $R^n \to R$, that has an associated weight vector $w = (w_1, w_2, \cdots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\text{IP-OWA}(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} \hat{p}_j a_{\sigma(j)}, \tag{12}$$

where: $\big(\sigma(1), \sigma(2), \cdots, \sigma(n)\big)$ is a permutation of $(1, 2, \cdots, n)$, such that $a_{\sigma(j-1)} \geq a_{\sigma(j)}$ for all $j = 2, \cdots, n$, p_j is the associated probability of $a_{\sigma(j)}$, and $\hat{p}_j = \dfrac{w_j p_j}{\sum\limits_{j=1}^{n} w_j p_j}$.

It's worth pointing out that IP-OWA operator is a good approach for unifying probabilities and OWAs in some particular situations. But it is not always useful, especially in situations where we want to give more importance to the probabilities or to the OWA operators. In order to show why this unification does not seem to be a final model, we could also consider other ways of representing $\hat{p}_j$. For example, we could also use $\hat{p}_j = \dfrac{w_j + p_j}{\sum\limits_{j=1}^{n}(w_j + p_j)}$ or other similar approaches.

Based on GOWA operator and IP-OWA operator, in the following, we shall develop the generalized immediate probability 2-tuple OWA (GIP-2TOWA) operator.

Definition 13. Let $\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple, An GIP-2TOWA operator of dimension is a mapping GIP-2TOWA: $S^n \to S$, that has an associated weight vector $w = (w_1, w_2, \cdots, w_n)^T$ such that $w_j > 0$ and $\sum\limits_{j=1}^{n} w_j = 1$. Furthermore,

$$\text{GIP-2TOWA}\big((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\big) = \Delta\left(\left(\sum_{j=1}^{n} \hat{p}_j \Delta^{-1}\big(r_{\sigma(j)}, a_{\sigma(j)}\big)^{\lambda}\right)^{1/\lambda}\right), \qquad (13)$$

where: $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $\big(r_{\sigma(j-1)}, a_{\sigma(j-1)}\big) \geq \big(r_{\sigma(j)}, a_{\sigma(j)}\big)$ for all $j = 2, \cdots, n$, each (r_i, a_i) has associated a probability p_i, p_j is the associated probability of $\big(r_{\sigma(j)}, a_{\sigma(j)}\big)$, and $\hat{p}_j = \dfrac{w_j p_j}{\sum\limits_{j=1}^{n} w_j p_j}$.

It's worth pointing out that GIP-2TOWA operator is a good approach for unifying probabilities and TOWAs in some particular situations. But it is not always useful, especially in situations where we want to give more importance to the probabilities or to the TOWA operators. In order to show why this unification does not seem to be a final model, we could also consider other ways of representing $\hat{p}_j$. For example, we could also use $\hat{p}_j = \dfrac{w_j + p_j}{\sum\limits_{j=1}^{n}(w_j + p_j)}$ or other similar approaches.

If we analyze different values of the parameter λ, we obtain another group of particular cases such as the usual IP-2TOWA operator, the IP-2TOWG operator, the IP-2TOWQA operator and the IP-2TOWHA operator.

Remark 9. When $\lambda = 1$, the GIP-2TOWA operator becomes the IP-2TOWA operator.

$$\text{IP-2TOWA}\big((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\big) = \Delta\left(\sum_{j=1}^{n} \hat{p}_j \Delta^{-1}\big(r_{\sigma(j)}, a_{\sigma(j)}\big)\right).$$

Note that if $p_j = 1/n$, for all (r_i, a_i), we get the 2TOWA operator and if $w_j = 1/n$, for all (r_i, a_i), we get the P-2TOWA operator.

Remark 10. When $\lambda = 0$, the GIP-2TOWA operator becomes the IP-2TOWG operator.

$$\text{IP-2TOW}G\big((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\big) = \Delta\left(\prod_{j=1}^{n}\Big(\Delta^{-1}\big(r_{\sigma(j)},a_{\sigma(j)}\big)\Big)^{\hat{p}_j}\right).$$

Note that if $p_j = 1/n$, for all (r_i, a_i), we get the 2TOWG operator and if $w_j = 1/n$, for all (r_i, a_i), we get the P-2TOWG operator.

Remark 11. When $\lambda = -1$, the GIP-2TOWA operator becomes the IP-2TOWHA operator.

$$\text{IP-2TOWHA}\big((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\big) = \Delta\left(\dfrac{1}{\displaystyle\sum_{j=1}^{n}\dfrac{\hat{p}_j}{\Delta^{-1}\big(r_{\sigma(j)},a_{\sigma(j)}\big)}}\right).$$

Note that if $p_j = 1/n$, for all (r_i, a_i), we get the 2TOWHA operator and if $w_j = 1/n$, for all (r_i, a_i), we get the P-2TOWHA operator.

Remark 12. When $\lambda = 2$, the GIP-2TOWA operator becomes the IP-2TOWQA operator.

$$\text{IP-2TOWQA}\big((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\big) = \Delta\left(\left(\sum_{j=1}^{n}\hat{p}_j\Delta^{-1}\big(r_{\sigma(j)},a_{\sigma(j)}\big)^2\right)^{1/2}\right).$$

Note that if $p_j = 1/n$, for all (r_i, a_i), we get the 2TOWQA operator and if $w_j = 1/n$, for all (r_i, a_i), we get the P-2TOWQA operator.

3. Illustrative example

In this section, we shall analyze a decision-making problem where a company is studying which strategy is the most appropriate for them (Merigó 2010). Because the environment is very uncertain, the experts in the company need to assess the available information by using linguistic information. Assume a company that operates in Europe and North America is analyzing its general policy for the next year. The board of directors has analyzed the economic situation of the company and they have found that now is a good moment to make an expansion policy to another continent in order to become more relevant in the world. They consider five possible strategies to follow: 1) A_1: Expand to the Asian market; 2) A_2: Expand to the African market; 3) A_3: Expand to the South American market; 4) A_4: Expand to all three continents; 5) A_5: Do not develop any expansion. In order to evaluate these strategies, the company considers that the key factor is the economic situation for the next year. Then, according to the situation, the expected benefits for the company will be different. The experts have considered five possible situations for the next year: 1) S_1: Negative growth rate;

2) S_2: Growth rate near 0; 3) S_3: Low growth rate; 4) S_4: Medium growth rate; 5) S_5: High growth rate. The five possible strategies A_i $(i = 1,2,3,4,5)$ are to be evaluated using the linguistic term set:

$$S = \{s_1 = extremely\ poor(EP), s_2 = very\ poor(VP), s_3 = poor(P), s_4 = medium(M),$$
$$s_5 = good(G), s_6 = very\ good(VG), s_7 = extremely\ good(EG)\}.$$

By the three decision makers under the above five situations. the expected results depending on the alternative A_i and the situation S_j are shown in Table 1.

Table 1. Available information about the strategies

	S_1	S_2	S_3	S_4	S_5
A_1	M	VP	EG	G	VG
A_2	EP	VP	P	VP	EP
A_3	VG	EG	M	VG	EG
A_4	M	G	VG	EP	M
A_5	G	EP	P	VP	VP

With respect to this problem, the experts in the company find probabilistic information given as follows: $P = (0.3, 0.3, 0.2, 0.1, 0.1)$. They assume that the WA, that represents the degree of importance of each state of situations, is: $\omega = (0.2, 0.25, 0.15, 0.3, 0.1)$. Note that the probabilistic information has an importance of 35% and the WA an importance of 65%. Furthermore, the policy of the company is to be very pessimistic in order to obtain safety results whenever the future results are not clear. Therefore, they decide to manipulate the probabilities by using the following OWA weighting vector: $w = (0.1, 0.2, 0.2, 0.0.2, 0.3)$. As we can see, this weighting vector seems to be conservative because it gives higher importance to the lowest result used in the last weight. Note that the company will use immediate probabilities in order to assess this problem. The results found in the immediate probabilities by using the above probabilities and weights are shown in Table 2.

Table 2. Immediate probabilities of the problem

	S_1	S_2	S_3	S_4	S_5
IP_1	0.286	0.429	0.095	0.095	0.095
IP_2	0.429	0.286	0.095	0.095	0.095
IP_3	0.316	0.158	0.316	0.105	0.105
IP_4	0.316	0.316	0.105	0.158	0.105
IP_5	0.150	0.450	0.200	0.100	0.100

Firstly, we use the GP-2TWA operator to aggregate the decision information in Table 1 and probabilistic information given as $P = (0.3, 0.3, 0.2, 0.1, 0.1)$. The results are shown in Table 3. The GP-2TWA operator includes the P-2TWA operator, P-2TWG operator, P-2TWHA operator and P-2TWQA operator.

Secondly, we use the GP-2TOWA operator to aggregate the decision information in Table 1 and probabilistic information given as $P = (0.3, 0.3, 0.2, 0.1, 0.1)$ and OWA weighting vector $W = (0.1, 0.2, 0.2, 0.0.2, 0.3)$. The results are shown in Table 4. The GP-2TOWA operator includes the P-2TOWA operator, P-2TOWG operator, P-2TOWHA operator and P-2TOWQA operator.

Thirdly, we use the GIP-2TOWA operator to aggregate the decision information in Table 1 and immediate probabilities information in Table 2. The results are shown in Table 5. The GIP-2TOWA operator includes the IP-2TOWA operator, IP-2TOWG operator, IP-2TOWHA operator and IP-2TOWQA operator.

Finally, according to the different aggregating operators, the ordering of the alternatives are shown in Table 6. Note that > means "preferred to". As we can see, depending on the aggregation operators used, the ordering of the strategies is slightly different. Therefore, depending on the aggregation operators used, the results may lead to different decisions. However, the best strategy is A_3.

Table 3. Linguistic aggregated results ($\beta = 0.35$)

	P-2TWA	P-2TWG	P-2TWHA	P-2TWQA
A_1	(M,0.40)	(M,0.00)	(M,-0.42)	(G,-0.27)
A_2	(VP,-0.17)	(VP,-0.30)	(VP,-0.44)	(VP,-0.04)
A_3	(VG,0.03)	(VG,-0.07)	(VG,-0.18)	(VG,0.12)
A_4	(M,-0.09)	(P,0.30)	(P,-0.47)	(M,0.28)
A_5	(P,-0.39)	(VP,0.21)	(VP,-0.13)	(P,-0.01)

Table 4. Linguistic aggregated results ($\beta = 0.35$)

	P-2TOWA	P-2TOWG	P-2TOWHA	P-2TOWQA
A_1	(M,0.30)	(M,-0.11)	(P,0.47)	(G,-0.35)
A_2	(VP,-0.33)	(VP,-0.47)	(EP,0.41)	(VP,-0.19)
A_3	(VG,-0.19)	(VG,-0.33)	(VG,-0.47)	(VG,-0.08)
A_4	(M,-0.18)	(P,0.24)	(P,-0.50)	(M,0.17)
A_5	(VP,0.41)	(VP,0.06)	(VP,-0.23)	(P,-0.23)

Table 5. Linguistic aggregated results

	IP-2TOWA	IP-2TOWG	IP-2TOWHA	IP-2TOWQA
A_1	(M,-0.29)	(P,0.33)	(P,-0.01)	(M,0.09)
A_2	(VP,-0.43)	(EP,0.45)	(EP,0.34)	(VP,-0.30)
A_3	(VG,-0.37)	(VG,-0.50)	(G,0.36)	(VG,-0.25)
A_4	(M,0.05)	(M,-0.40)	(P,-0.09)	(M,0.31)
A_5	(VP,0.20)	(VP,-0.18)	(VP,-0.45)	(P,-0.39)

Table 6. Ordering of the alternative

	Ordering
P-2TWA	$A_3 > A_1 > A_4 > A_5 > A_2$
P-2TWG	$A_3 > A_1 > A_4 > A_5 > A_2$
P-2TWHA	$A_3 > A_1 > A_4 > A_5 > A_2$
P-2TWQA	$A_3 > A_1 > A_4 > A_5 > A_2$
P-2TOWA	$A_3 > A_1 > A_4 > A_5 > A_2$
P-2TOWG	$A_3 > A_1 > A_4 > A_5 > A_2$
P-2TOWHA	$A_3 > A_1 > A_4 > A_5 > A_2$
P-2TOWQA	$A_3 > A_1 > A_4 > A_5 > A_2$
IP-2TOWA	$A_3 > A_4 > A_1 > A_5 > A_2$
IP-2TOWG	$A_3 > A_4 > A_1 > A_5 > A_2$
IP-2TOWHA	$A_3 > A_4 > A_1 > A_5 > A_2$
IP-2TOWQA	$A_3 > A_4 > A_1 > A_5 > A_2$

Conclusion

In this paper, we investigate the decision making problems by using probabilities, immediate probabilities and information that can be represented with linguistic labels and propose some new decision analysis. Firstly, we develop three new aggregation operators: generalized probabilistic 2-tuple weighted average (GP-2TWA) operator, generalized probabilistic 2-tuple ordered weighted average (GP-2TOWA) operator and generalized immediate probabilistic 2-tuple ordered weighted average (GIP-2TOWA) operator. These operators use the WA operator, the OWA operator, linguistic information, probabilistic information and immediate probabilistic information. They are quite useful because they can assess the uncertain information within the problem by using both linguistic labels and the probabilistic information that considers the attitudinal character of the decision maker. In these approaches, alternative appraisal values are calculated by the aggregation of 2-tuple linguistic information. Thus, the ranking of alternative or selection of the most desirable alternative(s) is obtained by the comparison of 2-tuple linguistic information. Finally, we give an illustrative example about selection of strategies to verify the developed approach and to demonstrate its feasibility and practicality. Theoretical analyses and numerical results all show that the method is straightforward and has no loss of information. In the future, we shall continue working in the application of the probabilistic aggregation operators with 2-tuple linguistic assessment information to other domains.

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