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MULTIMOORA OPTIMIZATION USED TO DECIDE ON A BANK LOAN TO BUY PROPERTY

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Abstract. Different multiple objectives are expressed in different units, which makes optimization difficult. Therefore the internal mechanical solution of a Ratio System, producing dimensionless numbers, is preferred to Weights which are most of the time used to compare the different units. In addition, the ratio system creates the opportunity to use a second approach: a non-subjective Reference Point Theory. The two approaches form a control on each other. The choice of the objectives is even more non-subjective if the opinion of all stakeholders interested in the issue is involved by the use of the Ameliorated Nominal Group and Delphi Techniques. The overall theory is called MOORA (Multi-Objective Optimization by Ratio Analysis). The results are still more convincing if a Full Multiplicative Form is added to MOORA under the name of MULTIMOORA. At that moment the control by three different approaches forms a guaranty for a solution being as non-subjective as possible. MULTIMOORA is used to decide upon a bank loan to buy property.

Keywords: MOORA, MULTIMOORA, bank loans, weights, Ratio System, Reference Point Theory, Full Multiplicative Form, Ameliorated Nominal Group and Delphi Techniques.

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JEL Classification: C44, C61, D22, E17, G21, M21.

The Problem

Just as an example: suppose a single person decides to buy a small apartment consisting of two rooms in, let us say, Lithuania. The cost price of the apartment is 30,025€. The person has sufficient income to pay the initial down payment including all kinds of fees and commissions and to guarantee the monthly loan payments, the payment of interests and of insurance.

The client is reluctant for the obligation to take a special life insurance. In addition he has to face the following obligations:

- The initial payment in €.
- Loan repayment and payment of interest in € per month.
- Hypothecation bond registration fee in €.
- A one off loan administration fee in €,
- Life insurance in € per year.
- Insurance of the apartment to be purchased in € per year.
- Commission for currency exchange in €.
- Monopolization of all bank activities and the wage administration of the client in % (0%, 50% or 100%)[1].

Even if two obligations are expressed in € the same € is not necessary meant. For instance, a Euro of initial payment can not be substituted by a Euro for the payment of interest.

We limit us to a simulation exercise. Contrary to a lot of other definitions, simulation is defined here in a rather broad sense. Gordon, Enzer and Rochberg (1970) give the most complete description of simulation as mechanical, metaphorical, game or mathematical analogs. They conclude: "are used where experimentation with an actual system is too costly, is morally impossible, or involves the study of problems which are so complex that analytical solution appears impractical".

Three banks are assumed in the simulation exercise, one foreign and two domestic banks A and B as shown in the following Table 1.

Table 1. Conditions for a Bank Loan of 30,025€

	initial payment	**monthly repayment**	**registration**	**administration**	**life insurance**	**insurance**	**exchange**	**bank deposits**
	MIN.	**MIN.**	**MIN.**	**MIN.**	**MIN.**	**MIN.**	**MIN.**	**MIN.**
Foreign Bank	**5617.6**	331.31	115.44	103.9	0	46.18	115.44	0
A	6926.41	317.46	129.87	115.44	69.26	57.72	0	0.5
B	6926.41	346.32	129.87	101.01	69.26	57.72	0	1

Which Method of Multi-Optimization is to be used for this Banking Activity?

MULTIMOORA is a strong instrument for solving the banking problem.

Table 1 (a matrix) assembles raw data with vertically numerous indicators, criteria or objectives and horizontally fictive banks.

[1] For these items Zavadskas *et al.* 2004 inspired this article. In addition we are grateful for a discussion with Prof. Banaitis, one of the other authors.

Two different methods under the name of MOORA, namely a Ratio System and a Reference Point Approach, will try to make optimal the content of the table. By adding the Full Multiplicative Form three methods will compose MULTIMOORA, whereas the three methods will control each other (for more information, see also: Brauers, Zavadskas 2010). The following diagram (Fig. I) clarifies the combination of the three different methods of MULTIMOORA (Balezentis *et al.* 2010).

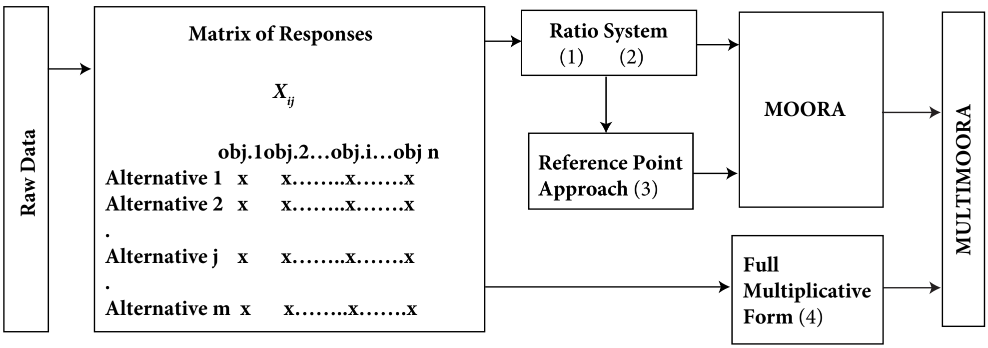

Fig. 1. Diagram of MULTIMOORA

The figures between brackets refer to the basic equations involved as shown in the following pages.

Ratio System of MOORA

We go for a ratio system in which each response of an alternative on an objective is compared to a denominator, which is representative for all alternatives concerning that objective[2]:

$$x_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum_{j=i}^{m} x_{ij}^{2}}}, \qquad (1)$$

with: x_{ij} = response of alternative j on objective I; $j = 1,2,...,m$; m the number of alternatives; $i = 1,2,…n$; n the number of objectives; x_{ij}^{*} = this time a dimensionless number representing the response of alternative j on objective i[3].

[2] Brauers and Zavadskas (2006) prove that the most robust choice for this denominator is the square root of the sum of squares of each alternative per objective.

[3] *Dimensionless Numbers,* having no specific unit of measurement, are obtained for instance by multiplication or division. The normalized responses of the alternatives on the objectives belong to the interval [0; 1]. However, sometimes the interval could be [-1; 1]. Indeed, for instance in the case of productivity growth some sectors, regions or countries may show a decrease instead of an increase in productivity i.e. a negative dimensionless number. Instead of a normal increase in productivity growth a decrease remains possible. At that moment the interval becomes [–1, 1]. Take the example of productivity, which has to increase (positive). Consequently, we look for a maximization of productivity

For optimization, these responses are added in case of maximization and subtracted in case of minimization:

$$y_j^* = \sum_{i=1}^{i=g} x_{ij}^* - \sum_{i=g+1}^{i=n} x_{ij}^*, \tag{2}$$

with: $i = 1,2,\ldots,g$ as the objectives to be maximized; $i = g+1, g+2,\ldots, n$ as the objectives to be minimized; y_j^* = the total assessment of alternative j with respect to all objectives; y_j^* can be positive or negative depending of the totals of its maxima and minima.

An ordinal ranking of the y_j^* shows the final preference. Indeed, cardinal scales can be compared in an ordinal ranking after Arrow (1974): "Obviously, a cardinal utility implies an ordinal preference but not *vice versa*".

The Reference Point Approach as a part of MOORA

The Reference Point Approach will go out from the ratios found in formula (1), whereby a Maximal Objective Reference Point is also deduced. The Maximal Objective Reference Point approach is called realistic and non-subjective as the co-ordinates (r_i), which are selected for the reference point, are realized in one of the candidate alternatives. In the example, A (10;100), B (100;20) and C (50;50), the maximal objective reference point R_m results in: (100;100). The Maximal Objective Vector is self-evident, if the alternatives are well defined, as for projects in Project Analysis and Project Planning.

Given the dimensionless number representing the normalized response of alternative j on objective i, namely x_{ij}^* of formula (1) and in this way arriving to:

$$\left(r_i - x_{ij}^*\right),$$

with: $i = 1, 2,\ldots, n$ as the attributes; $j = 1, 2,\ldots, m$ as the alternatives; r_i = the i^{th} co-ordinate of the reference point; x_{ij}^* = the normalized attribute i of alternative j then this matrix is subject to the *Metric of Tchebycheff* (Karlin and Studden 1966)[4]:

$$\min_{(j)} \left\{ \max_{(i)} \left| r_i - x_{ij}^* \right| \right\}, \tag{3}$$

$\left| r_i - x_{ij}^* \right|$ means the absolute value if x_{ij} is larger than r_i for instance by minimization.

Concerning the use of the maximal objective reference point approach as a part of MOORA some reserves can be made in connection with consumer sovereignty. Consumer sovereignty is measured with the community indifference locus map of the consumers (Brauers 2008: 92–94). Given its definition the maximal objective reference point can be pushed in the non-

e.g. in European and American countries. What if the opposite does occur? For instance, take the original transition from the USSR to Russia. Contrary to the other European countries productivity decreased. It means that in formula (1) the numerator for Russia was negative with the whole ratio becoming negative. Consequently, the interval changes to: [–1, +1] instead of [0, 1].

[4] Brauers (2008) proves that the Min-Max metric is the most robust choice between all the possible metrics of reference point theory.

allowed non-convex zone of the highest community indifference locus and will try to pull the highest ranked alternatives in the non-allowed non-convex zone too (Brauers, Zavadskas 2006: 460–461). Therefore an aspiration objective vector can be preferred, which moderates the aspirations by choosing smaller co-ordinates than in the maximal objective vector and consequently can be situated in the convex zone of the highest community indifference locus. Indeed stakeholders may be more moderate in their expectations. The co-ordinates q_i of an *aspiration objective vector* are formed as:

$$q_i < r_i,$$

$(r_i - q_i)$ being a subjective element we don't like to introduce subjectivity in that way again. Instead, a test shows that the min-max metric of Tchebycheff delivers points inside the convex zone of the highest community indifference locus (Brauers 2008: 98–103).

The Full Multiplicative Form

The following n-power form for multi-objectives is called from now on a *full-multiplicative form* in order to distinguish it from the mixed forms:

$$U_j = \prod_{i=1}^{n} x_{ij}, \tag{4}$$

with: $j = 1,2,...,m$; m the number of alternatives; $i = 1,2,\ldots,n$; n being the number of objectives; x_{ij} = response of alternative j on objective I; U_j = overall utility of alternative j.

The overall utilities (U_j), obtained by multiplication of different units of measurement, become dimensionless.

How is it possible to combine a minimization problem with the maximization of the other objectives? Therefore, the objectives to be minimized are denominators in the formula:

$$U_j^{'} = \frac{A_j}{B_j}, \tag{4´}$$

$$\text{with: } A_j = \prod_{g=1}^{i} x_{gi},$$

$j = 1,2,...,m$; m the number of alternatives; i = the number of objectives to be maximized,

$$\text{with: } B_j = \prod_{k=i+1}^{n} x_{kj},$$

n-i = the number of objectives to be minimized,
with: $U_j^{'}$ = the utility of alternative j with objectives to be maximized and objectives to be minimized.

If one of the $x_{ij} = 0$ it would mean that an objective is not present in an alternative. In that case a foregoing filtering stage can prescribe that an alternative with a missing objective to be an optimum is withdrawn from the beginning. Otherwise for the calculation of a maximum the zero factor is just left out.

A zero in a minimization problem is much more complicated. A real zero factor, like in the case of the absence of pollution, has to maintain its influence. Therefore the zero factor will receive an extremely low symbolic value like 0.01. If the zero factor means missing information then the situation is different and will ask for a serious correction. A correction factor has to be introduced being a bit larger than all corresponding factors of the other alternatives, for instance next ten, next hundred etc. With factors 8 and 11 next ten will be 20. With factors 80 and 110 next hundred will be 200 etc.

MULTIMOORA

MULTIMOORA is composed of MOORA and of the Full Multiplicative Form of Multiple Objectives and in this way up till now no other approach is known including three or more methods, in this way MULTIMOORA becomes the most robust system of multiple objectives optimization

The Importance given to an Objective

The method of multiple objectives which uses non-subjective dimensionless measures without normalization like in MULTIMOORA is more robust than this one which uses for normalization subjective weights (weights were already introduced by Churchman *et al.* in 1954 and 1957) or subjective non-additive scores like in the traditional reference point theory (Brauers 2004: 158–159).

The Additive Weighting Procedure (MacCrimmon 1968: 29–33), which was called SAW, Simple Additive Weighting Method by Hwang and Yoon (1981: 99) starts from the following formula:

$$\max . U_j = w_1 x_{1j} + w_2 x_{2j} + \ldots + w_i x_{ij} + \ldots + w_n x_{nj}, \tag{5}$$

U_j = overall utility of alternative j with j = 1, 2,….., m, m the number of alternatives; w_i = weight of attribute i indicates as well as normalization as the level of importance of an objective

$$\sum_{i=1}^{i=n} w_i = 1,$$

i = 1, 2,…, n; n the number of attributes and objective; x_{ij} = response of alternative j on attribute i.

In addition, weights adding up to one create a new artificial super-objective, denying any form of multiple objectivity.

Reference Point Theory is non linear with, this time, non-additive scores replacing weights. The non-additive scores take care of normalization.

With weights and scores importance of objectives is mixed with normalization. Indeed weights and scores are mixtures of normalization of different units and of importance coefficients. On the contrary the dimensionless measures of MULTIMOORA do not need external normalization. However the problem of importance remains.

In the Ratio System to give more importance to an objective its response on an alternative under the form of a dimensionless number could be multiplied with a Significance Coefficient:

$$\ddot{y}_j^* = \sum_{i=1}^{i=g} s_i x_{ij}^* - \sum_{i=g+1}^{i=n} s_i x_{ij}^*, \qquad (6)$$

with: i = 1, 2,…, g as the objectives to be maximized; i = g+1, g+2,…, n as the objectives to be minimized; s_i = the significance coefficient of objective I; $\ddot{y}_j^*$ = the total assessment with significance coefficients of alternative j with respect to all objectives.

For the Reference Point Approach the formula would be:

$$\left| s_i r_i - s_i x_{ij}^* \right|. \qquad (7)$$

If the Full Multiplicative Form has to stress the importance of an objective a α-term is added or an exponent is allocated on condition that it is done with unanimity or at least with a strong convergence in opinion of all stakeholders concerned.

As the stakeholders concern in the banking example only a group of candidate mortgagees a representative sample of them will be sufficient.

The attribution of sub-objectives represents another solution. Take the example of the purchase of fighter planes (Brauers 2002). For economics, the objectives concerning the fighter planes are threefold: price, employment and balance of payments, but there is also military effectiveness. In order to give more importance to military defense, effectiveness is broken down in, for instance, the maximum speed, the power of the engines and the maximum range of the plane. Anyway, the attribution method is more refined than that a significance coefficient method could be as the attribution method succeeds in characterizing an objective better. For instance, for employment two sub-objectives replace a significance coefficient of two and in this way characterize the direct and indirect side of employment. In the banking example for instance some criteria could be split for the older or for the younger generations.

In addition the problem is raised of the subjective choice of objectives in general, or could we call it robustness of a choice? The Ameliorated Nominal Group Technique, as explained in Brauers (2004: 44–60), will gather a representative sample of all the candidate mortgagees to determine the objectives in a non-subjective and anonymous way. The original Nominal Group Technique of Van De Ven and Delbecq (1971) was less robust as the Ameliorated Version, as this one excludes subjective wishes of the group. Indeed in the Ameliorated Nominal Group Technique the group is questioned about the probability of occurrence of an event. In this way the experts become more critical even about their own ideas. The probability of the group is found as the median of the individual probabilities. Finally, the group rating (R) is multiplied with the group probability (P) in order to obtain the effectiveness rate of the event (E). The events are translated into objectives and selected in a robust way by the Ameliorated Nominal Group Technique (for an example, see Brauers. Lepkova (2003), also Brauers, Zavadskas (2010: 18–19).

Given the absence of a representative sample of all mortgagees in the banking simulation all criteria are considered to have the same importance.

Be careful with Rank Correlation in MULTIMOORA

If we would use for MULTIMOORA the total of the ranks of the ratio system, the reference point and the multiplicative form at that moment we would work ordinal and arrive in the rank correlation method (Kendall 1948). The most robust multi-objective method has to satisfy the following condition: the method of multiple objectives based on cardinal numbers is more robust than this one based on ordinal numbers. "An ordinal number is one that indicates order or position in a series, like first, second, etc." (Kendall and Gibbons 1990: 1). Robustness of cardinal numbers is based first on the saying of Arrow (1974: 256): "Obviously, a cardinal utility implies an ordinal preference but not vice versa" and second on the fact that the four essential operations of arithmetic: adding, subtracting, multiplication and division are only reserved for cardinal numbers (see Brauers 2007 and also: Brauers and Ginevičius 2009: 137–138).

Axioms on Ordinal and Cardinal Scales

1. *A deduction of an Ordinal Scale, a ranking, from cardinal data is always possible* (Arrow).
2. *An Ordinal Scale can never produce a series of cardinal numbers* (Arrow).
3. *An Ordinal Scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.*

In application of axiom 3 we shall translate the ordinal scale of the three methods of MULTIMOORA in another one based on *Dominance*, *being Dominated, Transitivity and Equability*.

Dominance, being Dominated, Transitiveness and Equability

The three methods of MULTIMOORA are assumed to have the same importance. Stakeholders or their representatives like experts may have a different importance in ranking but this is not the case with the three methods of MULTIMOORA. These three methods represent all existing methods with dimensionless measures in multi-objective optimization and all the three have the same important significance.

Dominance

Absolute Dominance means that an alternative, solution or project is dominating in ranking all other alternatives, solutions or projects which are all being dominated. This absolute dominance shows as rankings for MULTIMOORA: (1-1-1).

General Dominance in two of the three methods is of the form with a < b < c <d:

(d-a-a) is generally dominating (c-b-b);
(a-d-a) is generally dominating (b-c-b);
(a-a-d) is generally dominating (b-b-c),

and further transitiveness plays fully.

Transitiveness

If *a* dominates *b* and *b* dominates *c* than also *a* will dominate *c*.

Overall Dominance of one alternative on another

For instance (a-a-a) is overall dominating (b-b-b) which is overall being dominated.

Equability

Absolute Equability has for instance the form: (e-e-e) for 2 alternatives.

Partial Equability of 2 on 3 exists e. g. (5-e-7) and (6-e-3).

Circular Reasoning

Despite all distinctions in classification some contradictions remain possible in a kind of *Circular Reasoning*.

We can cite the case of:

Object A (11-20-14) dominates generally object B. (14-16-15);

Object B. (14-16-15) dominates generally Object C (15-19-12);

but Object C (15-19-12) dominates generally Object A (11-20-14).

In such a case the same ranking is given to the three objects.

Application on the Banking Example

The different criteria as shown in table 1 are grouped in the following way:

1) The Initial Payment:
- The initial payment;
- Hypothecation bond registration fee;
- A one off loan administration fee;
- Commission for currency exchange.

2) Regular Payments:
- Loan repayment and payment of interest;
- Insurance of the apartment to be purchased.

3) Life Insurance.

4) Monopolization of all bank activities and the wage administration for the client.

Following Table 2 shows the new grouping.

Annexes A and B gives the details of the MULTIMOORA calculations. Table 3 shows the results.

The Foreign Bank shows an ABSOLUTE DOMINANCE on the other banks.

Bank A shows a GENERAL DOMINANCE OF TWO ON THREE MULTIMOORA rankings against Bank B.

Table 2. Criteria for Loan Payment

	1+3+4+7	2+6/12	5	8
	MIN.	MIN.	MIN.	
FB	5952.38	335.5	0	0
A	7171.72	322.27	69.26	0.5
B	7157.29	351.13	69.26	1

Table 3. MULTIMOORA results for the Banking Example

	RS	RP	MF	MULTIMOORA
FB	1	1	1	1
A	2	2	3	2
B	3	3	2	3

Conclusion

MOORA and MULTIMOORA present strong instruments to measure an optimum in economic calculus. MOORA is composed of a Ratio System, producing dimensionless numbers. In addition, the ratio system creates the opportunity to use its ratios as a starting point for a second approach: a non-subjective Reference Point Theory. The two approaches form a control on each other. The choice of the objectives is even more non-subjective if the opinion of all stakeholders interested in the issue are involved by the use of the Ameliorated Nominal Group and Delphi Techniques. The overall theory is called MOORA (Multi-Objective Optimization by Ratio Analysis). The results are still even more convincing when a Full Multiplicative Form is added to MOORA under the name of MULTIMOORA.

However at that moment we are in the ordinal sphere, the combination of three rankings. Which is the next step as summation is not allowed? Indeed addition belongs to the cardinal and not to the ordinal sphere. The following axioms are rather to be respected:

1. *A deduction of an Ordinal Scale, a ranking, from cardinal data is always possible.*
2. *An Ordinal Scale can never produce a series of cardinal numbers.*
3. *An Ordinal Scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.*

On the one side we have three separate rankings belonging to the ordinal sphere and on the other side we will replace them by another ranking decided by a theory around dominance.

Absolute Dominance means that an alternative, solution or project is dominating in ranking all other alternatives, solutions or projects which are all being dominated. This absolute dominance shows as rankings for MULTIMOORA: (1-1-1).

General Dominance in two of the three methods is of the form:

(d-a-a) is generally dominating (c-b-b);
(a-d-a) is generally dominating (b-c-b);
(a-a-d) is generally dominating (b-b-c).

Whereas **Transitiveness** means that if *a* dominates *b* and *b* dominates *c* then also *a* will dominate *c*.

Applied on the loan demand for an apartment from a bank in a simulation exercise a Foreign Bank shows an ABSOLUTE DOMINANCE on the other banks. A second Bank shows a GENERAL DOMINANCE OF TWO ON THREE MULTIMOORA rankings against a third Bank.

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Annex A

Table 4. MOORA applied on 3 banks Lt with 8 conditions

	initial payment	monthly repayment	registration	Admini-stration	life insurance	insurance	exchange	bank deposits			
	MIN.	MIN.	MIN.	MIN.	MIN.	MIN.	MIN.	MIN.			
4a – Matrix of Responses of Alternatives on Objectives: (x_{ij})											
FB	5617.6	331.31	115.44	103.9	0	46.18	115.44	0			
A	6926.41	317.46	129.87	115.44	69.26	57.72	0	0.5			
B	6926.41	346.32	129.87	101.01	69.26	57.72	0	1			
4b – Sum of squares and their square roots											
FB	31557429.76	109766.3161	13326.3936	10795.21	0	2132.5924	13326.3936	0			
A	47975155.49	100780.8516	16866.2169	13326.394	4796.9476	3331.5984	0	0.25			
B	47975155.49	119937.5424	16866.2169	10203.020	4796.9476	3331.5984	0	1			
Σ	127507740.7	330484.7101	47058.82740	34324.624	9594	8795.789	13326.394	1.2500			
root	11291.93255	574.8779958	216.9304667	185.26906	97.948431	93.785869	115.44	1.118034			
4c – Objectives divided by their square roots and MOORA									sum		
FB	0.497487917	0.576313587	0.532152084	0.5608060	0	0.4923983	1	0.000	-2.6591579	a_1	1
A	0.61339456	0.552221519	0.598671095	0.6230938	0.7071068	0.6154445	0	0.4472136	-1.8485051	a_2	2
B	0.61339456	0.602423475	0.598671095	0.5452071	0.7071068	0.6154445	0	0.8944272	-1.3736068	a_3	3
4d – Reference Point Theory with Ratios: co-ordinates of the reference point equal to the maximal objective values											
r_i	0.4975	0.5522	0.5322	0.5452	0	0.4923983	0	0.000			
4e – Reference Point Theory: Deviations from the reference point									max.	min.	
FB	0.0000	0.0241	0.0000	0.0156	0	0.0000	1	0.000	1.0000000	a_1	3
A	0.1159	0.0000	0.0665	0.0779	0.7071068	0.1230	0	0.4472136	0.7071068	a_2	1
B	0.1159	0.0502	0.0665	0.0000	0.7071068	0.1230	0	0.894	0.8944272	a_3	2

Table 4bis

4a´ - Matrix of Responses of Alternatives on Objectives: (x_{ij})				
	1+3+4+7	**2+6/12**	5	8
	MIN	MIN.	MIN.	
FB	5952.38	335.158333	0	0
A	7171.72	322.27	69.26	0.5
B	7157.29	351.13	69.26	1

4b´ – Sum of squares and their square roots				
FB	35430827.66	112331.1084	0	0
A	51433567.76	103857.9529	4796.9476	0.25
B	51226800.14	123292.2769	4796.9476	1
Σ	138091195.6	339481.3382	9593.89520	1.2500
root	11751.22102	582.6502709	97.94843133	1.118034

4c´ – Objectives divided by their square roots and MOORA						
					sum	min.
FB	0.506532895	0.575230717	0	0	1.0817636	**1**
A	0.610295729	0.55311053	0.707106781	0.4472136	2.3177266	**2**
B	0.609067771	0.602642816	0.707106781	0.8944272	2.8132446	3

4d´ – Reference Point Theory with Ratios: co-ordinates of the reference point equal to the maximal objective values						
r_i	0.5065	0.5531	0.0000	0.0000		

4e´ – Reference Point Theory: Deviations from the reference point						
					max	min.
FB	0.0000	0.0221	0.0000	0.0000	0.0221202	**1**
A	0.1038	0.0000	0.7071	0.4472	0.7071068	**2**
B	0.1025	0.0495	0.7071	0.8944	0.8944272	**3**

Annex B

Table 5. Full multplicative method applied on 3 banks Lt with 8 conditions

	1			3		4		5
	MIN.	**MIN.**		**MIN.**		**MIN.**		**MIN.**
FB	5617.6	331.31	16.955721	115.44	0.146879	103.9	0.0014137	100.0
A	6926.41	317.46	21.818213	129.87	0.168000	115.44	0.0014553	69.26
B	6926.41	346.32	20.000029	129.87	0.154000	101.01	0.0015246	69.26

	6		7		8		
	MIN.		**MIN.**		**MIN.**		
1.41366E-05	46.18	3.06119E-07	115.44	2.65176E-09	2	1.3259E-09	1
2.10122E-05	57.72	3.64037E-07	200	1.82018E-09	0.5	3.6404E-09	3
2.20128E-05	57.72	3.81372E-07	200	1.90686E-09	1	1.9069E-09	2

4a – Matrix of Responses of Alternatives on Objectives: (x_{ij})

	1	2	3	4	5	6	7	8
	initial payment	**monthly repay**	**registration**	**admini-stration**	**life insu-rance**	**insurance**	**exchange**	**deposit**
	MIN.	**MIN.**	**MIN.**	**MIN.**	**MIN.**	**MIN.**	**MIN.**	**MIN.**
FB	5617.6	331.31	115.44	103.9	0	46.18	115.44	2
A	6926.41	317.46	129.87	115.44	69.26	57.72	0	0.5
B	6926.41	346.32	129.87	101.01	69.26	57.72	0	1

Table 5bis. Full multplicative method applied on 3 banks Lt with 4 conditions

	3+4+7+1	2+6/12		5			8 and results	
	MIN.	**MIN.**		**MIN.**		**MIN.**		
FB	4952.38	335.1583333	14.77624009	100	0.1477624	2	0.0738812	1
A	7171.72	322.27	22.25376237	69.26	0.3213076	0.5	0.64261514	3
B	7096.68	351.13	20.21097599	69.26	0.2918131	1	0.29181311	2

SUMMARY OF THE 3 METHODS OF MULTIMOORA

	RS	RP	MF	MULTIMOORA	
FB	1	1	1	1	ABSOLUTE DOMINANCE
A	2	2	3	2	GENERAL DOMINANCE OF TWO ON THREE RANKINGS
B	3	3	2	3	

OPTIMIZAVIMAS MULTIMOORA METODU IMANT BANKO PASKOLĄ NEKILNOJAMAM TURTUI PIRKTI

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Santrauka. Įvairūs suinteresuotų asmenų tikslai išreiškiami įvairiais vienetais, o tai optimizavimą daro sudėtingą. Todėl skirtingiems vienetams palyginti reikalinga speciali įvertinimo sistema, kuri dydžius paverčia bedimensiais. Straipsnyje atliekamas optimizavimas pagal MULTIMOORA metodą imant banko paskolą nekilnojamajam turtui įsigyti.

Reikšminiai žodžiai: reikšmingumai, įvertinimo sistema, atskaitos taško teorija, MOORA, MULTI-MOORA, Delphi metodai.

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