




THE IMPORTANCE OF ENTERPRISE RISK MANAGEMENT IN LARGE COMPANIES IN COLOMBIA

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Abstract. The main aim of this paper is to establish that essential aspects are determinant in the enterprise risk identification (ERI) and the existing interrelationship between each corporate risk goals, in the large companies of Colombia. Study proposes a parametric analysis and a non-parametric. The first uses correlation matrices for statistical analysis, a multiple linear regression statistical tools to identify that essential aspects are determinant in the (ERI). The second proposes a new aggregation called the Bonferroni Induced Ordered Weighted Average Adequacy Coefficient (BON-IHOWAAC) operator and Bonferroni Induced Ordered Weighted Average the Maximum and Minimum level (BON-IHOWAIMAM) to establish the existing interrelationship between each corporate risk goals using the risk management information and manager perception. Of the results obtained is highlighted that for all economic sectors; first, control measures are highlighted in the (ERI) and second, the goal with the greatest interrelationship for the other ones to be achieved is protect people. Finally, the study concludes with a holistic analysis of the importance that executive team gives to the management of risks from the prioritization of objectives and the use of tools for the treatment of information to improve the process of decisions-making in uncertain contexts.

Keywords: enterprise risk management, aggregation operators, decision-making, Bonferroni OWA.

JEL Classification: C12, C14, C43, D81, L21, M10.

Introduction

Nowadays, globalization has taken over the world, causing great changes, generating organizations to confront them, regardless of their economic activity or size. The world economy is exposed to different risks where large companies interact with factors such as technology, legal and regulatory regulations, changing markets and competition. This interaction brings

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with it, a series of threats that can affect the attainment of organizational goals and benefits, preventing their permanence in the market. This situation that impacts companies has led to the need to develop strategies that help mitigate the uncertainty that generate risks and affect decision making.

To address the threats that surround them with greater force and speed, companies have adopted risk management as a strategy. Risk management allows more general and structured support companies to prevent unexpected events that may affect them, in order to respond to them and protect their resources (Mejía, Núñez, & Martinz, 2017). Thus, this has led to an integrated approach to measuring and managing risks known as Enterprise Risk Management (ERM) (Quon, Zeghal, & Maingot, 2012). ERM is a process that manages all risks in an integrated and holistic manner by controlling and coordinating compensation risks throughout the company (Thomas, Berry, & Jianren, 2018). Therefore, ERM has become an important model for the management of large companies, as it gives awareness of possible threats, deals with uncertainty in an effective manner, identifies and manages the associated risks and opportunities in order to meet the strategic goals of the organization and remain in the market. However, there is an environment in a constant evolution and full of uncertain facts that difficult business management. These uncertain facts cannot be placed in time and space, since the information provided by the past is limited for the forecast of these events (Kaufmann & Gil Aluja, 1986). Hence, one of the goals that companies have is to implement strategies that will help them mitigate the effects that lead to a future fraught with uncertainty, and avoid extinction. Colombian companies are also exposed to these conditions of risk and uncertainty of the environment, which has led them to adopt systems and standards to identify and mitigate their effects. However, the implementation of these systems are not efficient, since on the part of the managers of the companies they do not have clarity about the rules and lack sufficient information about the threats and weaknesses of the organization, and the benefits that are generated by implementing an irrigation identification and prevention system (Mejía, 2017). Thus, identifying risks is of great importance because it allows managers to make better decisions and also enables the organization to maintain a good performance and acquire competitive advantages over its competition.

Hence, The main aim of this paper is to establish that essential aspects are determinant in the enterprise risk identification and the existing interrelationship between each corporate risk goals, in the large companies of Colombia. Thus, a parametric analysis and a non-parametric analysis are proposed. The first uses correlation matrices for statistical analysis, a multiple linear regression statistical tools to identify that essential aspects are determinant in the enterprise risk identification. The second proposes a new aggregation operators combining BON-IOWA operator with some distance measures, which are called the Bonferroni Induced Ordered Weighted Average Adequacy Coefficient (BON-IHOWAAC) operator and Bonferroni Induced Ordered Weighted Average the Maximum and Minimum level (BON-IHOWAIMAM). The main advantage of these new operators is that can calculate the differences between two elements or two sets and reorder the arguments using order-inducing variables in order to solve more complex problems. Based on these methods, we have established the existing interrelationship between each corporate risk goals using the risk management information and perception of the managers. Of the results obtained is highlighted that for all economic

sectors. First, control measures (CM) are highlighted because the businessmen use control measures tools that allow preventing and minimizing the identified risks. Second, the goal with the greatest interrelationship for the other ones to be achieved is protect people. Furthermore, it is noted that for each economic sector there is a variation of the goals with less alienation according to expectations of senior managers who lead ERM, i.e. results depends on the perception of the manager and not on the type of information.

It has showed that statistical methods could be efficient in the analysis and prediction of events. However, in many cases, it is necessary to resort to other techniques that can also help managers to make decisions based on their preferences in an orderly and logical manner. Also, it is noteworthy that the results allow reviewing objective and subjective information from different sources obtained representative values, which provide a global view of the environment in these sectors. This study gives way to new research aimed at analyzing various methods, which provide a new holistic view of how enterprise risks are managed, through the perceptions and opinions of managers in different economic sectors at the national and international levels generating new scientific knowledge. Finally, This paper is structured as follows: In Section 1 theoretical framework is presented, which is form by uncertainty and decision-making and enterprise risk management. In Section 2, we defined parametric and non-parametric models. Likewise, new aggregation method is presented. In Section 3, results and mathematical application are explained. Finally, we present the conclusion and implications of the research.

1. Theoretical framework

Firms are exposed to different environmental changes, due to factors that generate uncertainty and risk in the development and achieve of its goals, which affect its management. Thus, an adequate management methodology to face uncertainty and risk can become in competitive advantage allowing the firm to grow and maintain over the time by itself. Therefore, it is necessary to analyze the main concepts of uncertainty and decision-making and enterprise risk management (ERM) in order to show how they can be combined and applied in decision-making process to improve business management in conditions of risk and uncertainty.

1.1. Uncertainty and decision making

Nowadays, business environment is in caothic and uncertain events where firms have to face multiple aggrissions, which are in a constant evolution caused by globalization. These events increase the difficulty of predicting social, economic and business phenomena. In fact, these events are dificult to forecast using information provided by the past. In fact, in the business environment, traditional prediction techniques are the most used to forecast the near future based on past information. However, these techniques do not take into account diffuse elements that contemplate soft and subjective aspects. Hence, companies face the challenge of using different methodologies that allow them to implement strategies to mitigate the effects of the environment. In this sense, researches have developed methodologies related to random and uncertainty to obtain sequences and significant scenarios of events. On the

one hand, random is understood as the measurement of observable events repeated in time, which are linked to the probability¹ concept. On the other hand, uncertainty occurs when there is not evidence of previous experiences where natural language and reasoning have a key role, which are related to the possibility² concept. Since the probabilistic methodologies are those accepted and used in the analysis of information, it is pertinent to study new methodological approaches that include uncertain and diffuse aspects.

The study of uncertainty is developed through fuzzy logic (Zadeh, 1965) providing new mathematical approximations. Within this field, decision-making studies have been carried out based on fuzzy sets concept (Zadeh, 1965), fuzzy environments (Bellman & Zadeh, 1970), the approximate reasoning (Zadeh, 1975a, 1975b, 1975c), aggregation operators (Yager, 1988), averaging functions (Beliakov, Bustince Sola, & Calvo Sánchez, 2016) and so on. These studies is based on the fact that a large number of decisions in the real world occur in an environment in which the consequences of possible actions are not known with precision. Thus, decision making process is configured by multiple stages with a high influence of human subjectivity (Blanco-Mesa, Merigó, & Gil-Lafuente, 2017). Besides, a decision is given within a multi-stage process where human intelligence has the ability to manipulate fuzzy concepts and respond to fuzzy instructions. Furthermore, a decision is an organizational activity of thought, in which intuition and logic are combined, i.e. it is also related to the probability and possibility concepts. Hence, multiple formal mathematical models have been developed from reason (soft) and logic (hard). Mathematical models help the mechanisms of logic, which in most cases have support, in formal or probabilistic theories. Hard formal models take into account certain data, probabilistic models, statistically measurable data or constructed from laws of probability (Kaufmann & Gil Aluja, 1986). Likewise, statistical models have proven to be efficient in the analysis and prediction of events. However, in many cases, it is necessary to resort to other softer techniques, in which the opinions and judgment of experts can be included for decision-making (Casanovas, Merigó, & Torres-Martínez, 2014).

There are many soft models that have been developed, among which the OWA operator proposed by Yager (1988) having a great interest in the scientific community. This operator is traditionally used to choose an alternative in situations of uncertainty, in which, in addition to not knowing the final result, it is not possible to predict its objective probability (Casanovas et al., 2014). Furthermore, this allows solving the problem for decision making and satisfy most of those involved through a process of aggregation of information that is not resolved in a simple way, since there may be criteria or opinions that have greater relevance (Casanovas et al., 2014). In this sense, the aggregation operator OWA (Yager, 1993, 2003a; Yager, Kacprzyk, & Beliakov, 2011) and their different extensions as induced variables (Merigó & Gil-Lafuente, 2009; Yager & Filev, 1999; Yager, 2003b), distance measures (Merigó & Gil-Lafuente, 2008, 2012; Merigó & Yager, 2013), Pythagoreans (Yager, 2014), means (Liu, Mao, Zhang, & Li, 2013; Modave, Ceberio, & Kreinovich, 2008; Yager, 2008, 2009), linguistics (Herrera & Herrera-Viedma, 2000; Herrera, Herrera-Viedma, & Verdegay, 1996; Her-

¹ Probability in decisions is related to random, randomness, measurement and objective that are supported by a set of reality for reality with past events. (Blanco-Mesa, 2015).

² Possibility in decisions are related to the fuzzy, the valuations, the subjective, the perception and the sensation, where the realization of the facts can not be located in time and space (Blanco-Mesa, 2015).

ra & Martinez, 2000), uncertain (Xu & Da, 2002), probability (Merigó, 2010; Merigó & Wei, 2011), intuitionistic (Zeng, Su, & Zhang, 2016), heavy (Yager, 2002) among others, they offer multiple alternatives that allow a better solution to different sort of problems, since they are the different sort of information. Each decision maker can aggregate the information in a different way according to his or her degree of optimism and pessimism from a series of data, to a single representative value of the information can be obtained (Merigó, 2014). One of the most important advantages of these operators is the great flexibility they have to include among them other concepts used to make decisions or assess the opinions of experts (Casanovas et al., 2014).

Recently, operators have been developed that combine several methods that generate more complete extensions allowing the treatment of more complex problems. Among these we can highlight the combination of the Bonferroni means (Bonferroni, 1950) and OWA operator (Yager, 1988) obtaining Bonferroni OWA operator (Beliakov, James, Mordelová, Rückschlossová, & Yager, 2010; Yager, 2009). According to (Beliakov et al., 2016) Bonferroni means is a aggregation function composed of two arithmetic means and the product, i.e., it involves the product of each argument with the average of the rest of the arguments and OWA operator is class of averaging aggregation functions composed of a weighted arithmetic means in that the weights are associated with their magnitude, i.e., all inputs are equivalent, and the importance of an input is determined by its value. Hence, BON-OWA operator allows making a multiple comparison and interrelating all inputs continuously.

This operator has generated great interest in the scientific community, which has led to the development of various extensions where other methods are combined with the operator BON-OWA and Bonferroni means. Among these are the operators that combine distance measures (Blanco-Mesa & Merigó, 2017; Blanco-Mesa & Merigo-Lindahl, 2016; Blanco-Mesa, Merigó, & Kacprzyk, 2016; Merigó, Palacios-Marqués, & Soto-Acosta, 2017), linguistic variables (Dutta & Guha, 2015; Jiang & Wei, 2014; Liu, Rong, Chu, & Li, 2014), induced variables (Blanco-Mesa, León-Castro, & Merigó, 2018a; Blanco-Mesa, León-Castro, Merigó, & Xu, 2018b) uncertainty variables (Wei, Zhao, Lin, & Wang, 2013; Xu, 2012), intuitionistic – membership and non-membership functions– (Dutta & Guha, 2013; Y. He & Z. He, 2015; Xu & Yager, 2011), geometric mean (Gong, Hu, Zhang, Liu, & Deng, 2015; D. Li, Zeng, & J. Li, 2016; Xia, Xu, & Zhu, 2013), hesitant fuzzy sets (Wang, Li, Zhou, & Yang, 2014; Yu, Wu, & Zhou, 2012; Zhu & Xu, 2013; Zhu, Xu, & Xia, 2012) and among others, which have focused on the treatment of decision problems demonstrating their usefulness and utility by combining the characteristics of several operators into one. Thus, it can be seen that the extensions with the Bonferroni average and the OWA operator have a great potential for development that can lead to future applications in real environments.

1.2. Enterprise risk management

The increase in volatility in the business world has revealed the inadequacy of traditional but fragmented approaches to risk management. This has led to an integrated approach to measuring and managing risks known as Enterprise Risk Management (ERM) (Quon et al., 2012). ERM is a process that manages all risks in an integrated and holistic manner by controlling and coordinating compensation risks throughout the company (Thomas et al., 2018).

According to Wu, Olson, and Dolgui (2015) ERM is the most modern approach to managing the risks an organization faces from a system perspective. Likewise, ERM is considered a discipline that allows companies in any economic sector to evaluate, leverage, finance, and control different risks to increase the value of the organization, protect resources, preserve a good image and ensure the survival of the company in the time (Casualty Actuarial Society, 2003). Thus, ERM is not only responsible for reducing threats and hazards, but also turning them into opportunities and benefits for the organization (Bill, 2006).

The ERM is conceived as a conglomerate of standards that are related and integrated to each other achieving that business risks have an effective management (ISO, 2009b). These standards provide guidelines on how risk management should be carried out; both at the general level and specifically with the type of economic sector to which all types of companies belong. Furthermore, these standards requires the alignment of risk management with corporate governance and strategy (Bromiley, McShane, Nair, & Rustambekov, 2015). Corporate governance for risk management is understood as the establishment of principles, guidelines, policies, strategies and good practices for managing risks (OECD, 2014). This portfolio of tools allow policy makers continue to focus on mechanisms improving corporate governance and risk management (Beasley, Clune, & Hermanson, 2005). In addition, it helps to control the management, since it mitigates the concern of the stakeholders for fraud, the need for greater transparency, the disclosure of the company to the market, and the increased accountability by boards of directors (Demidenko & McNutt, 2010). In this sense, the ERM is a structured proposal that integrates strategy, processes, people, technology and knowledge in order to reduce the uncertainty that produces the environment that the company must face as it remains in the market. Thus, ERM has arisen to be perceived as a new form of strategic business management relating the business strategy to the daily risks of the organization (Bill, 2006).

In this sense, ERM allows transforming the organization as a whole, by involving elements such as risk governance, practices, tools, communication, consultation and the development of risk management. These elements are fundamental to obtain a good risk management system that generates a good organizational culture improving decision making (IRM & Protiviti, 2012). The *risk government* designs integrated structures that define how the business risk management system is implemented within the organization (Bill, 2006). This entails that companies incorporate this mechanism that helps them foresee and control all kinds of threats generated by the environment. Likewise, risk management provides *tools* that involve different stages throughout its processes, which in a coordinated manner seek to direct, control and evaluate the actions of an organization with respect to the management of all types of risks. In addition, a continuous and consistent procedure and a follow-up cycle are required (Deloitte, Touche, & IMEF, 2003; ISO, 2009a). Monitoring begins with the identification and evaluation of risks, which leads to identifying them collectively and evaluating them according to their probability of occurrence. Then, they are classified grouping and prioritizing the risks by activity or within a dependency. Finally, they are quantified by helping the organization understand the potential impact of these risks (Bill, 2006). To execute each one of the stages, there are practices that allow the analysis of the strategic context and the risk assessment in order to monitor the existing risks to generate the necessary alerts and the companies can mitigate and control the risks (AS/NZS-4360, 2004; ISO, 2009a).

Similarly, *communication and consultation* is a fundamental and constant process of organizations, from which information is provided and obtained in a dialogue with stakeholders who have some link with the company (ISO, 2009b). The information generated is taken into account for the decision-making process both internally and externally (ISO, 2009b). The communication and consultation mechanisms are used to create trust in the organization and to guarantee a better understanding of the critical risks to which the company is exposed (HB 327, 2010). These mechanisms are supported by communication channels, which should be designed with appropriate technology, common language and concepts, to ensure that all work teams understand the objectives and vision of the risk strategy (Bill, 2006). As a result of the communication process, information for risk management and effective management of information security requires a coordinated effort at all levels of the organization. The risk management framework (RMF) process is emphasized in the core tasks in individual information systems to obtain and maintain security clearance and provide cost-effective protection for information assets in line with the risk to the organization operating their systems (Gantz & Philpott, 2013). Likewise, within communication and consultation mechanisms the cultural aspects are determinant in the development of a risk culture. Risk culture is a term that describes the values, beliefs, knowledge and understanding of risk shared by a group of people with a common goal, in particular the employees of an organization or of teams or groups within an organization (Davidson, Mackenzie, Wilkinson, & Burke, 2015). Thus, risk culture is applicable to any organization, but its implementation will differ widely from one context to another.

Finally, a structured and systematic way of highlighting effective aspects of the risk management processes, based on the priorities, is the level of maturity of the *development of risk management* (RIMS, 2006). This measurement is made from different scales, which allow a comparative evaluation of the best practices of risk management in an organization. Likewise, this allows increasing and strengthening the capacity that companies must have to manage risks (Hillson, 1997). ERM involves holistic decision making for critical corporate functions, such as capital budgeting and management (Ai, Brockett, & Wang, 2017). Therefore, a generic approach is necessary to assess the ERM by using well-known frameworks and methodologies that are divided into the operational phase of an ERM and the measurement of the output of an ERM. The first phase is mainly based on standardized methodologies and includes a maturity model to assess the maturity of ERM implementation. The second phase demonstrates the importance of using multiple key risk indicators (KRI) at the strategic level to identify the outcome of the ERM (Kopia, Just, Geldmacher, & Bubian, 2017). A risk indicator provides a forward direction of information about the risk, which may or may not exist and is used as a warning system for future actions. A KRI allows monitoring a specific risk to carry out mitigation actions. The metrics are used to provide an early warning signal for greater exposure to risk in different aspects of the company (Scarlat, Chirita, & Bradea, 2012). Thus, ERM system reaches the development of its maturity when organizations understand their strategy, identify, measure, classify and monitor their risks, make decisions on which require more administrative attention generating a positive impact on the organization by increasing its value (Bill, 2006).

1.3. Enterprise risk management in Colombia

With the globalization of the economy, Colombian companies are more sensitive to social, political, economic and environmental changes, which increases their exposure to different risks. In this sense, there is a need to manage irrigation and control the threats that affect the efficient functioning and generate economic, social, environmental and corporate image losses (Mejía, 2013). Thus, companies have included in their organizational processes, risk management as a policy that allows the identification and risk assessment mechanisms to reduce the frequency and severity of them, generating internal control systems that become the path that points out the fulfillment of his missionary goals (Correa, Rios, & Acevedo, 2017). ERM encompasses activities and strategies that allow the company to identify, measure, reduce or systematically exploit, control and monitor exposure to various types of corporate risks. When considering the interactive effects of different risk events, the ERM offers a balance between the dual nature of risk, ensuring effective protection against threats and leverage opportunities (Marc, Sprcic, & Žagar, 2018).

Based on the above, some private and public institutions have proposed some systems and standards for risk management in an integral manner. In this sense, standards such as NTC 31000, NTC-OHSAS 18001 and NTC-ISO 14001 have been established, which the state adopted to determine guidelines, valuation methods, and assessment tools for business risk management. Also, Committee of Sponsoring Organizations of the Treadway Commission (COSO) has led and promoted methodologies and guidelines for business risk management and internal control, in order to improve business performance and reduce fraud in organizations. The methodologies developed are the COSO I in 1992, the COSO II methodology in 2004 and the COSO III methodology in 2017. These methodologies provide a conceptual framework of internal control with the fundamental objective of integrating the various definitions and concepts. Likewise, they form the control framework oriented towards risk management in companies with an integrating approach and the creation of benefits for stakeholders through techniques such as the administration of a risk portfolio (COSO, 2004). In addition, it demonstrates how the integration of risk management practices of the entire company helps accelerate growth and improve performance (COSO, 2017).

These systems and standards have led companies to integrate these regulations in their management processes in order to develop better business practices and mitigate and manage risk in a better way. In a study conducted by (Cifuentes et al., 2017), it is shown that the large private companies in Bogota have continuous improvement, due to the implementation of an integrated risk system, a positive use of tools and methodologies and the assurance in the continuity of organizational operations. However, it reflects an aspect to improve by the managers of companies to not know or be clear about these standards, as it derives problems by lacking information on external threats and internal weaknesses of the organization and the benefits that are generated by implementing an irrigation identification and prevention system (Mejía, 2017). Thus, large Colombian companies have difficulties in the identification and evaluation of risks (Mejía, 2017), since they are unaware of the importance of identifying, rating, evaluating, treating and monitoring risks, allowing unfavorable events to deteriorate the organization. In this sense, properly applied risk management will bring benefits, being self-sufficient in the resources used to increase efficiency and operational ef-

iciency (ICONTEC, 2009); improve learning, culture and organizational flexibility (Arnold, Benford, Canada, & Sutton, 2015). Hence, identifying risks is of great importance, since it allows managers to make better decisions and also enables the organization to maintain a good performance and acquire competitive advantages over its competition.

2. Methodology

In this work, we have analyzed the perceptions that have CEO's in large firms in Colombia about ERM using parametric and nonparametric models. Parametric analysis determines different variables association relates to existing tools for identifying and minimizing risks, commitment of senior management and specific management area. Nonparametric analysis shows the degree of importance of the goals and their interrelationship with expected financial benefits and the sorts of risks in ERM. To develop this study, we have taken the database that collects information related to ERM in large companies in Colombia. This database is composed of qualitative and quantitative information obtained from the 300 large companies, which are supplied by Chamber of Commerce of Bogotá (CCB). Based on this list, the study sample is determined using random sampling (where has 10% error rate and confidence level of 95%) obtaining as a result a total of 86 companies belonging to the extraction, manufacturing, services and commerce sectors. Companies are contacted by telephone to invite them and answer a questionnaire via online. As an instrument for gathering information, a questionnaire is used that included dichotomous, multiple response and liker scale inquiries. This questionnaire is conform by four major practical sections, development, communication and governance in risk management, which respond to the main aspects in ERM (COSO, 2017). Finally, the level of the answers surpasses the sample initially established, where the response rate on the population is 43%, obtaining a total of 129 companies surveyed, from which the respective analyses are carried out.

2.1. Parametric model

To carry out the parametric analysis, the four essential elements in business risk management; tools, governance, communication and development, have been taken into account in order to determine the study variables. These have been selected under the criterion of possible response to the importance that large companies in Colombia give to the identification of business risks to improve their management. As a result, four (4) variables per element were taken into account for a total of sixteen (16) study variables. Based on the above, hypotheses were constructed in order to explain the tools that companies used to manage and minimize risks. Likewise, it seeks to expose the commitment that managers have with the management of risks and the provision of areas for their management. In order to validate these statements, hypothesis tests are performed using correlation matrices, obtaining proportionality and the relationship that exists between the different variables. Subsequently, a multiple linear regression model is proposed to estimate the dependency ratio of the identification of risks at the corporate level related to the monitoring of risks for decision making, control measures and resources allocated to risk management. This model is expressed as follows:

$$RI = \beta_0 + \beta_1 MRDM + \beta_2 CM + \beta_3 RA + \varepsilon_i, \tag{1}$$

where *RI* is the risk identification (risks in business units, areas, processes, projects, products or services at the corporate level), β_0 is the constant, *MRDM* is the regressor variable of the use of risk monitoring for decision making by senior management, *MC* is the regressor variable of the application of control measures for the assessment risks, *RG* is the regressor variable of the necessary resources for the management of risks (financial, human, infrastructure, technological, among others).

2.2. Nonparametric model

Nonparametric model looks for to find the degree of importance between goals of risk management by each economic sector by means of new mathematical proposition related to aggregation theory and distance measures. These new propositions allow aggregating information within the minimum and the maximum values in order to parameterize the pessimistic and optimistic attitude of the decision maker (Blanco-Mesa & Merigó, 2016; Blanco-Mesa et al., 2016). Thus, it is proposed a new aggregation method using induced ordered weighted average (IOWA) operator (Merigó & Casanovas, 2011), Bonferroni OWA operator (Yager, 2009) and the adequacy coefficient (Kaufmann & Gil Aluja, 1987) and the index of maximum and minimum level (Gil-Lafuente, 2002).

2.3. Basic concepts of distance measures, IOWA and Bonferroni OWA operators

This section presented some basic concepts that have been used throughout the paper, including the Induced Ordered Weighted Average (IOWA) operator, Bonferroni operators and distance operators.

The Hamming distance (Hamming, 1950) is a useful technique for calculating the differences between two elements, two sets, etc. the weighted Hamming distance can be defined as follows.

Definition 1. A weighted Hamming distance of dimension *n* is a mapping $d_{WH} : R^n \times R^n \rightarrow R$ that has an associated weighing vector *W* of dimension *n* with the sum of the weights being 1 and $w_j \in [0,1]$ such that:

$$d_{WH} (\langle x_1, y \rangle_1, \dots, \langle x_n, y_n \rangle) = \sum_{j=1}^n w_j |x_i - y_i|, \tag{2}$$

where x_i and y_i are the *i*th arguments of the sets *X* and *Y*.

The adequacy coefficient (Kaufmann & Gil Aluja, 1987) is very similar to the Hamming distance with the difference that it neutralizes the result when the comparison shows that the real element is higher than that of the ideal one. Adequacy coefficient can be defined as follows.

Definition 2. A weighted adequacy coefficient of dimension *n* is a mapping $K : [0,1]^n \times [0,1]^n \rightarrow [0,1]$ that has an associated weighting vector *W* of dimension *n* with the sum of the weights 1 and $w_j \in [0,1]$ such that:

$$K(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_{i=1}^n w_i [1 \wedge (1 - x_i + y_i)], \tag{3}$$

where x_i and y_i are the i th arguments of the sets X and Y .

The index of maximum and minimum level is an index that unifies the Hamming distance and the adequacy coefficient in the same formulation (Gil-Lafuente, 2002). The index of maximum and minimum level can be defined as follows.

Definition 3. An AWIMAM of dimension n is a mapping $K: [0,1]^n \times [0,1]^n \rightarrow [0,1]$ that has an associated weighting vector W of dimension n with the sum of the weights 1 and $w_i \in [0,1]$ such that:

$$\eta(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \sum_u Z_i(u) * |x_i(u) - y_i(u)| + \sum_v Z_i(v) * [0 \vee x_i(v) - y_i(v)], \tag{4}$$

where x_i and y_i are the i th arguments of the sets X and Y .

Definition 4. An IOWA operator of dimension n is an application $IOWA: R^n \times R^n \rightarrow R$ that has a weighting vector associated, W of dimension n where the sum of the weights is 1 and $w_j \in [0,1]$, where an induced set of ordering variables are included (u_i) such that the formula is:

$$IOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{j=1}^n w_j b_j, \tag{5}$$

where b_j is the a_i value of the OWA pair $\langle u_i, a_i \rangle$ having the j th largest u_i . u_i is the order inducing variable and a_i is the argument variable.

Also note that if using order-inducing variables does the reordering step the OWAD operator becomes the Induced OWAD (IOWAD) operator. The definition is as follows (Merigó & Casanovas, 2011):

Definition 5. An IOWAD operator of dimension n is a mapping $IOWAD: R^n \times R^n \times R^n \rightarrow R$ that has an associated weighting vector W such that $w_j \in [0,1]$ and $W = \sum_{j=1}^n w_j = 1$, according to the following formula:

$$IOWAD(\langle u_1, x_1, y_1 \rangle, \dots, \langle u_n, x_n, y_n \rangle) = \sum_{j=1}^n w_j b_j, \tag{6}$$

where b_j is the $|x_1 - y_1|$ value of the IOWAD triplet $\langle u_i, x_i, y_i \rangle$ having the j th largest u_i . u_i is the order-inducing variable and $|x_1 - y_1|$ is the argument variable represented in the form of individual distances.

Definition 6. The Bonferroni OWA is a mean type aggregation operator. It can be defined by using the following expression:

$$BON - OWA(a_1, \dots, a_n) = \left(\frac{1}{n} \sum_i a_i^r OWA_W(V^i) \right)^{\frac{1}{r+q}}, \tag{7}$$

where (V^i) is the vector of all a_j except a_i . Let W be an OWA weighing vector of dimension $n - 1$ with components $w_i \in [0,1]$ when $\sum_i w_i = 1$. Then, we can define this aggregation as

$OWA_W(V^i) = \left(\sum_{j=1}^{n-1} w_j a_{\pi_k(j)} \right)$, where $a_{\pi_k(j)}$ is the largest element in the $n - 1$ tuple $V^i = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$.

Note that if $w_i = \frac{1}{n-1}$ for all i , $OWA_W(V^i) = \left(\frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n a_j^q \right)$, and Eq. (7) becomes the classical Bonferroni mean.

Definition 7. A BON-OWAD distance for two sets $X = \{x_1, x_2, \dots, x_n\}$ and $Y = \{y_1, y_2, \dots, y_n\}$ is given by:

$$BON - OWAD(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \left(\frac{1}{n} \sum_i D_i^r OWAD_{w_i}(V^i) \right)^{\frac{1}{r+q}}, \tag{8}$$

where (V^i) is the vector of all $|x_j - y_j|$ except $|x_i - y_i|$. Let W be an OWA weighing vector of dimension $n - 1$ with components $w_i \in [0, 1]$ when $\sum_i w_i = 1$. Then, we can define this aggregation as $OWAD_W(V^i) = \left(\sum_{j=1}^{n-1} w_j D_{\pi_k(j)} \right)$, where $D_{\pi_k(j)}$ is the largest element in the $n - 1$ tuple $V^i = (\langle x_1, y_1 \rangle, \dots, \langle x_{i-1}, y_{i-1} \rangle, \langle x_{i+1}, y_{i+1} \rangle, \dots, \langle x_n, y_n \rangle)$.

Note that if $w_i = \frac{1}{n-1}$ for all i , $OWAD_{w_i}(V^i) = \left(\frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n D_j^q \right)$, and Eq. (8) becomes Bonferroni mean distance (Blanco-Mesa et al., 2016).

2.4. New methods to Bonferroni IOWA operator with, the adequacy coefficient and the index of maximum and minimum levels

In the literature there are researches that have presented extensions that combine the BON-OWA and distance measures such as, Bonferroni IOWA operator (Blanco-Mesa et al., 2018b), Bonferroni IHOWA operator (Blanco-Mesa et al., 2018a), Bonferroni OWA Distance (BON-OWAD) (Blanco-Mesa & Merigo-Lindahl, 2016; Merigó et al., 2017), Bonferroni OWA Adequacy Coefficient BON-OWAAC, Bonferroni OWA the Maximum and Minimum level (BON-OWAIMAM) (Blanco-Mesa & Merigó, 2016; Blanco-Mesa et al., 2016), Bonferroni OWA Weighted Average Distance (BON-OWAWAD) (Blanco-Mesa et al., 2016; Merigó et al., 2017), Bonferroni Hybrid Weighted Distance (BON-HWD) and Bonferroni Immediate Weighted OWA Distance BON-IWOWAD (Blanco-Mesa & Merigó, 2017). Thus, in this section is presented a new extension of the BON-IOWA operato with some distance measures. In this new operator the characteristics of the IOWA operator (Merigó & Casanovas, 2011) is combined with adequacy coefficient (Kaufmann & Gil Aluja, 1987) and the maximum and minimum level index (Gil-Lafuente, 2002) in two formulation, calling they the Bonferroni Induced Ordered Weighted Average Adequacy Coefficient (BON-IHOWAAC) operator and Bonferroni Induced Ordered Weighted Average the Maximum and Minimum level (BON-IHOWAIMAM). The main advantage of these new operators is that can calculate the differences between two elements or two sets and reorder the arguments using order-inducing variables in order to solve more complex problems.

Proposition 1. A BON-IOWAAC adequacy coefficient for two sets $X = \{x_1, x_2, \dots, x_n\}$ and $Y = \{y_1, y_2, \dots, y_n\}$ such that

$$BON - IOWAAC(\langle u_1, x_1, y_1 \rangle, \dots, \langle u_n, x_n, y_n \rangle) = \left(\frac{1}{n} \sum_i AC_i^r IOWAAC_{w_i}(V^i) \right)^{\frac{1}{r+q}}, \quad (9)$$

where $IOWAAC_{w_i}(V^i) = \left(\frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n AC_j^q \right)$ with (V^i) being the vector of all $1 \wedge (1 - x_j + y_j)$

except $1 \wedge (1 - x_i + y_i)$ and w_i being an $n - 1$ vector W_i associated with α_i whose components w_{ij} are the IOWAAC weights, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable.

Let W be an OWA weighing vector of dimension $n - 1$ with components $w_i \in [0,1]$ when $\sum_i w_i = 1$, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable. Then, we can define this aggregation as $IOWAAC_W(V^i) = \left(\sum_{j=1}^{n-1} w_i AC_{\pi_k(j)} \right)$,

where $AC_{\pi_k(j)}$ is the largest element in the $n - 1$ tuple $V^i = V^i =$. Note that if $w_i = \frac{1}{n-1}$

for all i , $IOWAAC_{w_i}(V^i) = \left(\frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n AC_j^q \right)$, and Eq. (9) becomes Bonferroni induced mean adequacy coefficient.

Furthermore, BON-IHOWAAC has the following properties:

- 1) $BON - IOWAAC^{r,q}(0, 0, \dots, 0) = 0$;
- 2) $BON - IOWAAC^{r,q}(ac, ac, \dots, ac) = ac$, if $ac_k = ac$, for all k ;
- 3) $BON - IOWAAC^{r,q}(ac_1, ac_2, \dots, ac_n) \geq BON - IOWAAC^{r,q}(ad, ad, \dots, ad)$, i.e., $BON - IOWAAC^{r,q}$ is monotonic, if $ac_k \geq ad_k$, for all k ;
- 4) $\max_k \{ac_k\} \leq BON - IOWAAC^{r,q}(ac_1, ac_2, \dots, ac_n) \leq \min \{ac_k\}$.

In addition, if $q = 0$, then $BON - IOWAAC^{r,0}(ac_1, ac_2, \dots, ac_n) = \left(\frac{1}{n} \sum_{k=1}^n AC_k^r \right)^{1/r}$. If

$r = 2$ and $q = 0$, then BON-IOWAAC reduces to square mean adequacy coefficient:

$$BON - IOWAAC^{r,0}(ac_1, ac_2, \dots, ac_n) = \left(\frac{1}{n} \sum_{k=1}^n AC_k^2 \right)^{1/2}.$$

If $r = 1$ and $q = 0$, then BON-IOWAAC reduces to average distance:

$BON - IOWAAC^{r,0}(ac_1, ac_2, \dots, ac_n) = \frac{1}{n} \sum_{k=1}^n AC_k$. If $r \rightarrow +\infty$ and $q = 0$, then BON-IOWAAC

reduces to the max operator: $\lim_{r \rightarrow +\infty} BON - IOWAAC^{r,0}(ac_1, ac_2, \dots, ac_n) = \max \{AC_k\}$, if

$r \rightarrow 0$ and $q = 0$, then BON-IOWAAC reduces to geometric mean adequacy coefficient:

$$\lim_{r \rightarrow 0} BON - IOWAAC^{r,0}(ac_1, ac_2, \dots, ac_n) = \left(\prod_{k=1}^n AC_k \right)^{1/n}.$$

If $r = q = 1$, then BON-IOWAAC reduces to the following expression:

$$BON - IOWAAC^{1,1}(ac_1, ac_2, \dots, ac_n) = \left(\frac{1}{n(n-1)} \right) \sum_{\substack{k,j=1 \\ k \neq j}}^n AC_k AC_j .$$

Example 1. We have assumed that BON-IOWAC pair's $\langle \mu_i, x_i, y_i \rangle$ is given by $X = (0.3, 0.4, 0.7, 0.8)$, $Y = (0.1, 0.5, 0.8, 0.3)$ and induced variable are $U = (4, 7, 2, 6)$. W^* is the weighting vector of the $\langle \mu_i, x_i, y_i \rangle$ associated with α_i whose components v_{ij} . Here we shall let W^* , instead of being 1, it is 1 and it has the following values: $W^* = 0.2, 0.4, 0.1, 0.3$. The ordered OWA pair's is $\langle 7, (1 \wedge (1 - 0.4 + 0.5)) \rangle, \langle 6, (1 \wedge (1 - 0.8 + 0.3)) \rangle, \langle 4, (1 \wedge (1 - 0.3 + 0.1)) \rangle, \langle 2, (1 \wedge (1 - 0.7 + 0.8)) \rangle$ that is the ordered list x_i, y_i is $(1 \wedge (1 - 0.4 + 0.5)), (1 \wedge (1 - 0.8 + 0.3)), (1 \wedge (1 - 0.3 + 0.1)), (1 \wedge (1 - 0.7 + 0.8))$. We take $r = q = 0.5$. In addition:

$$\begin{aligned} V^1 &= (1 \wedge (1 - 0.8 + 0.3)) + (1 \wedge (1 - 0.3 + 0.1)) + (1 \wedge (1 - 0.7 + 0.8)); \\ V^2 &= (1 \wedge (1 - 0.4 + 0.5)) + (1 \wedge (1 - 0.3 + 0.1)) + (1 \wedge (1 - 0.7 + 0.8)); \\ V^3 &= (1 \wedge (1 - 0.4 + 0.5)) + (1 \wedge (1 - 0.8 + 0.3)) + (1 \wedge (1 - 0.7 + 0.8)) \text{ and} \\ V^4 &= (1 \wedge (1 - 0.4 + 0.5)) + (1 \wedge (1 - 0.8 + 0.3)) + (1 \wedge (1 - 0.3 + 0.1)). \end{aligned}$$

Using this get:

$$\begin{aligned} IOWAAC_{v_1}(V^1) &= 0.1 \times ((1 \wedge (1 - 0.8 + 0.3)) + (1 \wedge (1 - 0.3 + 0.1)) + (1 \wedge (1 - 0.7 + 0.8))) = 0.23; \\ IOWAAC_{v_3}(V^3) &= 0.3 \times ((1 \wedge (1 - 0.4 + 0.5)) + (1 \wedge (1 - 0.8 + 0.3)) + (1 \wedge (1 - 0.7 + 0.8))) = 0.75; \\ IOWAAC_{v_4}(V^4) &= 0.4 \times ((1 \wedge (1 - 0.4 + 0.5)) + (1 \wedge (1 - 0.8 + 0.3)) + (1 \wedge (1 - 0.3 + 0.1))) = 0.92; \end{aligned}$$

$BON - IOWAAC =$

$$\left(\frac{1}{4} \times ((0.23 \times (1 \wedge (1 - 0.4 + 0.5))) + (0.56 \times (1 \wedge (1 - 0.8 + 0.3)))) + ((0.75 \times (1 \wedge (1 - 0.3 + 0.1))) + (0.92 \times (1 \wedge (1 - 0.7 + 0.8)))) \right)^{0.5} .$$

Proposition 2. A BON-IOWAIMAM the maximum and minimum level index for two sets $X = \{x_1, x_2, \dots, x_n\}$ and $Y = \{y_1, y_2, \dots, y_n\}$ such that

$$BON - IOWAIMAM(u_1, x_1, y_1, \dots, u_n, x_n, y_n) = \left(\frac{1}{n} \sum_i IMAM_i^r IOWAIMAM_{w_i}(V^i) \right)^{\frac{1}{r+q}}, \quad (10)$$

where $IOWAIMAM_{w_i}(V^i) = \left(\frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n IMAM_j^q \right)$ with (V^i) being the vector of all $[|x_j - y_j|$

and the $[0 \vee (x_j, y_j)]$ except $|x_i - y_i|$ and the $[0 \vee (x_i, y_i)]$ and w_i being an $n - 1$ vector W_i associated with α_i whose components w_{ij} are the IOWAIMAM weights, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable.

Let W be an OWA weighing vector of dimension $n - 1$ with components $w_i \in [0, 1]$ when $\sum_i w_i = 1$, where the weights are associated according to the largest value of

u_i and u_i is the order-inducing variable. Then, we can define this aggregation as

$$IOWAIMAM_W(V^i) = \left(\sum_{j=1}^{n-1} w_j IMAM_{\pi_k(j)} \right), \text{ where } IMAM_{\pi_k(j)} \text{ is the largest element in the}$$

$n - 1$ tuple $V^i = V^i = (\langle u_1, x_1, y_1 \rangle, \dots, \langle u_{i-1}, x_{i-1}, y_{i-1} \rangle, \langle u_{i+1}, x_{i+1}, y_{i+1} \rangle, \dots, \langle u_n, x_n, y_n \rangle)$.

Note that if $w_i = \frac{1}{n-1}$ for all i , $IOWAIMAM_{w_i}(V^i) = \left(\frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n IMAM_j^q \right)$, and Eq. (10)

becomes Bonferroni induced mean the maximum and minimum level index.

Furthermore, BON-IOWAIMAM as the following properties:

- 1) $BON - IOWAIMAM^{r,q}(0, 0, \dots, 0) = 0$;
- 2) $BON - IOWAIMAM^{r,q}(ac, ac, \dots, ac) = ac$, if $ac_k = ac$, for all k ;
- 3) $BON - IOWAIMAM^{r,q}(ac_1, ac_2, \dots, ac_n) \geq BON - IOWAIMAM^{r,q}(ad, ad, \dots, ad)$, i.e., $BON - IOWAIMAM^{r,q}$ is monotonic, if $ac_k \geq ad_k$, for all k ;
- 4) $\max_k \{ac_k\} \leq BON - IOWAIMAM^{r,q}(ac_1, ac_2, \dots, ac_n) \leq \min \{ac_k\}$.

In addition, if $q = 0$, then $BON - IOWAIMAM^{r,0}(ac_1, ac_2, \dots, ac_n) = \left(\frac{1}{n} \sum_{k=1}^n IMAM_k^r \right)^{1/r}$.

If $r = 2$ and $q = 0$, then BON-IOWAIMAM reduces to square mean adequacy coefficient:

$$BON - IOWAIMAM^{r,0}(ac_1, ac_2, \dots, ac_n) = \left(\frac{1}{n} \sum_{k=1}^n IMAM_k^2 \right)^{1/2}.$$

If $r = 1$ and $q = 0$, then BON-IOWAIMAM reduces to average the maximum and mini-

imum level: $BON - IOWAIMAM^{r,0}(ac_1, ac_2, \dots, ac_n) = \frac{1}{n} \sum_{k=1}^n IMAM_k$.

If $r \rightarrow \infty$ and $q = 0$, then BON-IOWAIMAM reduces to the max operator:

$$\lim_{r \rightarrow +\infty} BON - IOWAIMAM^{r,0}(ac_1, ac_2, \dots, ac_n) = \max \{IMAM_k\},$$

if $BON - IOWAIMAM^{r,0}(ac_1, ac_2, \dots, ac_n) = \frac{1}{n} \sum_{k=1}^n IMAM_k$.

If $r \rightarrow 0$ and $q = 0$, then BON-IOWAIMAM reduces to geometric mean the maximum

and minimum level: $\lim_{r \rightarrow 0} BON - IOWAIMAM^{r,0}(ac_1, ac_2, \dots, ac_n) = \left(\prod_{k=1}^n IMAM_k \right)^{1/n}$.

If $r = q = 1$, then BON-IOWAIMAM reduces to the following expression:

$$BON - IOWAIMAM^{1,1}(ac_1, ac_2, \dots, ac_n) = \left(\frac{1}{n(n-1)} \sum_{\substack{k,j=1 \\ k \neq j}}^n IMAM_k IMAM_j \right).$$

Example 1. We have assumed that BON-IOWAIMAM pair's $\langle \mu_i, x_i, y_i \rangle$ is given by $X = (0.3, 0.4, 0.7, 0.8)$, $Y = (0.1, 0.5, 0.8, 0.3)$ and induced variable are $U = (4, 7, 2, 6)$. W^* is the weighting vector of the $\langle \mu_i, x_i, y_i \rangle$ associated with α_i whose components v_{ij} . Here we shall let W^* , instead of being 1, it is 1 and it has the following values: $W^* = 0.2, 0.4, 0.1, 0.3$. The or-

dered OWA pair's is $\langle 7, (1 \wedge (1 - 0.4 + 0.5)) \rangle, \langle 6, |0.8 - 0.3| \rangle, \langle 4, (1 \wedge (1 - 0.3 + 0.1)) \rangle, \langle 2, |0.7 - 0.8| \rangle$ that is the ordered list x_i, y_i is $(1 \wedge (1 - 0.4 + 0.5)), |0.8 - 0.3|, (1 \wedge (1 - 0.3 + 0.1)), |0.7 - 0.8|$. We take $r = q = 0.5$. In addition:

$$V^1 = |0.8 - 0.3|, (1 \wedge (1 - 0.3 + 0.1)), |0.7 - 0.8|;$$

$$V^2 = (1 \wedge (1 - 0.4 + 0.5)) + (1 \wedge (1 - 0.3 + 0.1)) + |0.7 - 0.8|;$$

$$V^3 = (1 \wedge (1 - 0.4 + 0.5)) + |0.8 - 0.3| + |0.7 - 0.8| \text{ and}$$

$$V^4 = (1 \wedge (1 - 0.4 + 0.5)) + |0.8 - 0.3| + (1 \wedge (1 - 0.3 + 0.1)).$$

Using this get:

$$IOWAIMAM_{v_1}(V^1) = 0.1 \times (|0.8 - 0.3| + (1 \wedge (1 - 0.3 + 0.1)) + |0.7 - 0.8|) = 0.14;$$

$$IOWAIMAM_{v_2}(V^2) = 0.2 \times ((1 \wedge (1 - 0.4 + 0.5)) + (1 \wedge (1 - 0.3 + 0.1)) + |0.7 - 0.8|) = 0.38;$$

$$IOWAIMAM_{v_3}(V^3) = 0.3 \times ((1 \wedge (1 - 0.4 + 0.5)) + |0.8 - 0.3| + |0.7 - 0.8|) = 0.48;$$

$$IOWAIMAM_{v_4}(V^4) = 0.4 \times ((1 \wedge (1 - 0.4 + 0.5)) + |0.8 - 0.3| + (1 \wedge (1 - 0.3 + 0.1))) = 0.92;$$

$$BON - IOWAIMAM =$$

$$\left(\frac{1}{4} \times ((0.14 \times (1 \wedge (1 - 0.4 + 0.5))) + ((0.38 \times (|0.8 - 0.3|))) + ((0.48 \times (1 \wedge (1 - 0.3 + 0.1)))) + ((0.92 \times (|0.7 - 0.8|))) \right)^{0.5}.$$

2.4.1. Mathematical application

In this section, the steps of the mathematical application of the aforementioned operators are presented.

Step 1. Using the data set of business risk management (ERM) in large companies in the survey of Colombia, we have taken the main objective established in ERM, which are common for the four major sectors defined consumption, services, manufacturing and extractive (see Table 1). These are identified as G₁, G₂, G₃, G₄, G₅ and G₆.

Table 1. Main goal set in ERM

	Goals
G ₁	Guarantee continuity of operation
G ₂	Comply with internal and external rules
G ₃	Guarantee availability and quality of information
G ₄	Keep the good will
G ₅	Prevent economic losses
G ₆	Protect people

Step 2. Using the data set, the most relevant goals in each sector was determined according to the sort of risk (Strategic (S), Operational (O), Financial (F), Technological (T), Social (S), Environmental (E), Labor (L), Legal (Le), Physics (P) and Insurable (I)). Here, it is clear

that some risks are more important than others, depending on the sector (see Tables 2, 3, 4 and 5). With these data, we can make a continuous comparison and establish the distance relationship between each of the goals taking into account the different sorts of risk for each sector as characteristics in the same formulation.

Table 2. Trade and consumer sector

	S	O	F	T	S	E	L	Le	P	I
G ₁	0.176	0.321	0.286	0.071	0.036	0.107	0.071	0.143	0.036	0
G ₂	0.059	0.071	0.071	0	0	0	0.036	0	0	0
G ₃	0	0.071	0.071	0	0	0.036	0	0.036	0	0
G ₄	0.059	0.071	0.036	0.036	0.036	0.036	0.071	0	0	0
G ₅	0.353	0.107	0.214	0	0	0.071	0.107	0.036	0	0
G ₆	0	0	0.036	0	0	0.071	0.107	0.071	0.036	0

Table 3. Service sector

	S	O	F	T	S	E	L	Le	P	I
G ₁	0.1	0.229	0.157	0.1	0.014	0.129	0.143	0.157	0.010	0.029
G ₂	0.057	0.114	0.071	0.071	0	0.071	0.043	0.071	0.01	0
G ₃	0.043	0.043	0.029	0.029	0	0.014	0.057	0.029	0	0.014
G ₄	0.014	0.029	0.014	0.029	0	0.014	0.014	0	0.01	0
G ₅	0.029	0.114	0.114	0.029	0.029	0.043	0.029	0.029	0.01	0
G ₆	0.043	0.129	0.057	0	0.014	0.1	0.143	0.086	0.03	0

Table 4. Manufacturing sector

	S	O	F	T	S	E	L	Le	P	I
G ₁	0.000	0.235	0.176	0	0	0.059	0.059	0.176	0.000	0.059
G ₂	0	0.176	0.118	0	0	0.059	0.118	0.059	0	0
G ₃	0	0.118	0.118	0.059	0.118	0.059	0	0	0	0
G ₄	0	0	0	0	0	0	0	0	0	0
G ₅	0	0.118	0.059	0	0	0	0	0.059	0	0
G ₆	0	0.118	0.059	0	0	0.059	0.235	0.059	0	0.059

Table 5. Mining sector

	S	O	F	T	S	E	L	Le	P	I
G ₁	0.143	0.357	0.000	0.000	0.000	0.214	0.071	0.10	0.000	0.000
G ₂	0	0	0	0	0	0	0	0	0	0
G ₃	0	0.071	0	0	0	0.071	0.071	0	0	0
G ₄	0.071	0	0	0	0	0	0.071	0.1	0	0
G ₅	0	0	0	0	0	0.071	0.071	0	0	0
G ₆	0.071	0.286	0	0	0.071	0.357	0.214	0.1	0	0

Step 3. To make a technical comparison between each goal for each sector, we have used BON-IOWAAC and BON-IOWAIMAM. The main idea is to compare the objectives according to the sort of risks. In this sense, we have taken as a weighted vector the expected financial benefit (EB) (see Table 6).

Table 6. Weighted average of OWA operator

	W ₁	W ₂	W ₃	W ₄	W ₅	W ₆	W ₇	W ₈	W ₉	W ₁₀
EB	0.122	0.237	0.185	0.056	0.022	0.122	0.119	0.107	0.019	0.011
Order	6	1	2	7	8	4	3	5	9	10

Step 4. It is carried on continuous comparison between each goal using the maximum similarity sub-relations [40]. Hence, it is found a dissimilarity fuzzy relation [D] through its complement, where $[D] = |1 - [S]$. From [D] we obtain a symmetric and reflexive matrix. Finally, it is shown the importance of each goal according to and how the goals in ERM contribute to benefits in decision-making for each sector.

3. Results

The results show the analysis of the existing tools to identify the risks, how to minimize the areas of risk and the specific areas for risk management. In addition, a multiple linear regression model shows the estimation of the dependency ratio of the identification of risks at the corporate level. Finally, the level of importance between the objectives of risk management and the expected financial benefits by economic sector is sought.

3.1. Statistic analysis: correlation matrices and multiple regression

The results of the correlation matrices and the multiple linear regression are shown. The correlation matrices analyse four essential aspects in the ERM; tools for risk identification, commitment of top management with enterprise risks, tools that minimize enterprise risks and specific areas for risk management.

First, the variables on the tools used by large companies in Colombia for the identification of risks, the application of treatment measures, monitoring of risks and the use of risk maps for their management are examined. Initially, it was determined that 79.1% of large companies in Colombia have a very favorable perception regarding the identification of the risks carried out in business units, areas, processes, projects, products or services at the corporate level. In addition, 69.8% of enterprises apply risk treatment measures, using key indicators and corporate risk maps to monitor risks at a strategic and operational level. Hypothesis tests are performed through a correlation analysis between the different variables in this category to identify the sort of association between them. The hypotheses of the tool category are:

H0: the use of key indicators for the monitoring of strategic and operational risks in the organization does NOT count as an effective measure for the treatment of these.

H1: the use of key indicators for the monitoring of strategic and operational risks in the organization counts as an effective measure for the treatment of these.

In the correlation matrix, it is observed that the association between the variable “the effectiveness of the application of risk treatment measures” and the variable “key indicators for risk monitoring” have a strong positive correlation of 0.649 and a level of significance of 0.00 for which the null hypothesis is rejected and the alternative hypothesis is accepted (see Table 7). In this sense, the identification of risks within organizations, the establishment and use of key indicators are fundamental for their monitoring. In addition, there is also evidence of a strong relationship between risk indicators as an effective measure of application for the treatment of these. Likewise, it noteworthy that the use of risk maps has a moderate positive relationship with the identification of corporate risks. Therefore, the generation of indicators is essential to be able to make an assertive identification of the risks and generate information maps to carry out an effective treatment of them.

Table 7. Correlation matrix of the tools for risk identification

		Identification	Treatment	Maps	Indicators
Identification	R ²	1	0.596**	0.467**	0.671**
	Sig.		0	0	0
Treatment	R ²	0.596**	1	0.637**	0.649**
	Sig.	0		0	0
Maps	R ²	0.467**	0.637**	1	0.573**
	Sig.	0	0		0
Indicators	R ²	0.671**	0.649**	0.573**	1
	Sig.	0	0	0	

Note: ** The correlation is significant at the 0.01 level (bilateral).

Second, we examine the variables on who is responsible for the key decisions of risk management in the organization, the effective commitment of senior management to use the results of monitoring for decision making and communication of results to the parties involved. It is highlighted that 89.9% of large companies in Colombia have a very favorable perception that the key decisions of risk management depend on senior management having an effective commitment to risk management. Likewise, 69.7% of these companies use the results of risk monitoring to make decisions and communicate results and risk management guidelines to the parties involved. Thus, the following matrix of correlations arise in the following hypotheses:

H0: Monitoring the risks for decision-making is NOT related to the effective Commitment that Top Management has towards risk management.

H1: Monitoring the risks for decision-making is related to the effective Commitment that Top Management has towards risk management.

In the matrix of correlations (see Table 8), it is observed that the association between the variable “Monitoring of risks for decision making” and “Effective commitment of senior management towards risk management” have a correlation positive strong of 0.536 and a level of significance of 0.000 for which the null hypothesis is rejected and the alternative hypothesis is accepted. In this sense, it is highlighted that in risk monitoring process, com-

munication between parties involved and senior management commitment are essential to establish clear communication channels and carry out an effective commitment by risk managers within the organization. Likewise, it is observed that the monitoring of risks has a non-significant positive relationship for decision making by senior management. Thus, ERM not only depends on senior management, but also involves and integrates strategy, processes, people, technology and knowledge (Bill, 2006).

Table 8. Correlation matrix of the senior management commitment with enterprise risks

		Decision-making	Commitment	Monitoring	Comunication
Decision-making	R ²	1	0.281**	0.168	0.191*
	Sig.		0.001	0.057	0.03
Commitment	R ²	0.281**	1	0.536**	0.522**
	Sig.	0.001		0	0
Monitoring	R ²	0.168	0.536**	1	0.559**
	Sig.	0.057	0		0
Comunication	R ²	0.191*	0.522**	0.559**	1
	Sig.	0.03	0	0	

Note: ** The correlation is significant at the 0.01 level (bilateral). * The correlation is significant at the 0.05 level (bilateral).

Likewise, the variables on the integrated risk management system are examined, taking into account the risk event registers, the control measures and the evaluation of the risks identified for the improvement of the management. It is observed that 63.5% of large companies in Colombia have an advanced risk management system applying control measures for the risks evaluated in their organizations. In addition, 66.7% use registers of risk events to improve their management, evaluating all identified and qualified risks. The hypothesis test was carried out through the correlation analysis between the different variables of this category, which were:

H0: the registry of risk events for the improvement of management is NOT related as a control measure for the risks evaluated.

H1: the registry of risk events for the improvement of management is related as a control measure for the risks evaluated.

It is observed that the association between the variable “Control measures for the evaluated risks” and the variable “Registration of risk events for the improvement of management” have a strong positive correlation of 0.648 and a level of significance of 0.00 that the null hypothesis is rejected and the alternative hypothesis is accepted (see Table 9). In this sense, the registry of risk events is one of the most relevant tools to take control measures in the management of risks. Likewise, it is noteworthy that in organizations is necessary to implement risk management systems that act as a control measure to reduce the levels of uncertainty and prevent losses that affect their market value.

Table 9. Correlation matrix of the tools that minimize enterprise risks

		Sistems	Control	Registration
Sistems	R ²	1	0.465**	0.383**
	Sig.		0	0
Control	R ²	0.465**	1	0.648**
	Sig.	0		0
Registration	R ²	0.383**	0.648**	1
	Sig.	0	0	

Note: ** The correlation is significant at the 0.01 level (bilateral).

Finally, the variables on the culture of risk management and the existence of a specific area that coordinates risk management have been examined, in which it is taking into account the different points of view of the parties involved and also the necessary resources assigned for its management. Here, it is reflected that 72.9% of large companies in Colombia see the culture of risk management in organizations very favorable and that they also have an area, dependency, department or instance that coordinates risk management. Additionally, 74.4% of these allocate all the necessary resources for risk management, such as financial, human, infrastructure, among others. The hypotheses of the correlation matrix are the following:

H0: if all the resources necessary for risk management are allocated, it is because there is NO Risk Management Culture.

H1: if all the resources necessary for risk management are allocated, it is because there is a Culture of risk management.

Taking into account the matrix of correlations (see Table 10), it is observed that the association between the variable “Risk management culture” and the variable “all resources necessary for risk management are allocated” have a strong positive correlation of 0.621 and a level of significance of 0.00, for which the null hypothesis is rejected and the alternative hypothesis is accepted. In this sense, culture has a significant relationship with the necessary

Table 10. Correlation matrix of specific area to ERM

		Culture	Area	Involved	Resources
Culture	R ²	1	-0.317**	0.445**	0.621**
	Sig.		0	0	0
Area	R ²	-0.317**	1	-0.351**	-0.173*
	Sig.	0		0	0.05
Involved	R ²	0.445**	-0.351**	1	0.430**
	Sig.	0	0		0
Resources	R ²	0.621**	-0.173*	0.430**	1
	Sig.	0	0.05	0	

Note: ** The correlation is significant at the 0.01 level (bilateral). * The correlation is significant at the 0.05 level (bilateral).

resources that can be assigned to risk management, in turn, within this culture, the different points of view of the parties involved are taken into account, which allow defining the criteria for the identification, measurement, control and evaluation of risks. However, this culture is not influenced by the existence of a specific area that coordinates risk management within organizations.

Based on the results of the correlation matrices, the most representative variables are selected under the ratio criterion closest to 1 to establish the multiple regression model. Different models are designed to estimate the relationships and predictions between variables in order to understand which of the independent variables are related to the dependent variable.

The multiple linear regression model (see Table 11) seeks to estimate the dependency ratio of risk identification at the corporate level (dependent variable) related to risk monitoring for decision making (RMDM), control measures (CM) and resources allocated (RA) for risk management (independent variables). The first columns collect the value of the partial regression coefficients (B) and their typical error. Taking into account the values thrown by the non-standardized coefficient, the values of B will be used to solve the regression equation looking for the RI value, i.e. the exact value that is expected from the dependent variable according to the values of B that obtain the independent variables, the result is the following:

Table 11. Multiple linear regression of the dependent variable identification of enterprise risks

	Non-standardized coefficients		Standardized coefficients			Collinearity statistics		
	B	Desv. Error	Beta	t	Sig.	R ² Ajus.	Tolerance	VIF
(Constante)	0.988	0.281	0	3.521	0.001	0.519	0	0
RMDM	0.137	0.075	0.154	1.811	0.073		0.52	1.922
CM	0.518	0.077	0.557	6.689	0		0.541	1.847
RG	0.102	0.069	0.107	1.485	0.14		0.722	1.386

Within the statistics, the t-scores can be evidenced, indicating that the variables taken into account contribute significantly to the prediction model by being different from zero (0), i.e. that the values obtained can be generalized to the population. (t = 1.811, 6.689, 1.485, p < 0.001). Also, it is observed that the R² of the three independent variables explain 52% of the dependent variance (R² = 0.519). In addition, the values of the inflated variance factor (VIF) comply with the assumption of non-multicollinearity (values between 1.386 and 1.922), i.e. there are not perfectly linear relationships between the explanatory variables of the model and therefore, the estimators obtained and the precision of the variables are not affected. With these results, it can be confirmed that the proposed model explains the dependent variable and there is not variable that over, but that all the variables contribute significantly to the model. In the values of the standardized coefficient, the Beta provides an idea about the relative importance of each variable within the equation. The most important and most significant variable (Sig. = 0.000) for the model are the control measures (CM), because the businessmen take control measures such as the flowchart of processes, the diagnostic questionnaire, risk maps, the risk matrix, the record of events, among others. These are tools that allow preventing and minimizing the identified risks.

3.2. Non parametric analysis: BON-IOWAAC and BON-IOWAIMAM operators

In this section we can see the main results obtained from the mathematical application. It has used the methods proposed above: BON-IOWAAC and BON-IOWAIMAM. These methods allow aggregating, comparing, reordering and establishing relationship continuously the distance between each argument. Based on these methods, the risk management information and manager perceptions are used to compare goals in ERM through the different sorts of risks as main characteristics and expected financial benefit as weighted vectors for four main economic sectors (see Tables 12 and 13). Likewise, it is used a graphical representation to show the summary results matrix (see Figures 1 and 2). These figures represent the distance that exists between the goals to achieve a common goal for each sector.

In Figures 1 and 2 are shown the summary results of the BON-IOWAAC and the BON-IOWAIMAM operators for each economic sector. It is observed that for each economic sector the achievement of a goal is interrelated with the other five goals of risk management, i.e. there is a minimum distance between them. For BON-IOWAAC, commerce and consumer sector is noteworthy that goal G_6 there exists minimum difference, which indicates that there is a strong relationship between each goal to protect people. Also, in this sector is highlighted that goals are G_1 and G_2 there exists maximum difference between each goals, which means that there is a weak interrelationship between goals to guarantee continuity of operation comply with internal and external rules, and guarantee the availability and quality of information.

Table 12. Degree of important of goals matrix using BON-IOWAAC operator

	Trading and consumer sector						Service sector					
	G_1	G_2	G_3	G_4	G_5	G_6	G_1	G_2	G_3	G_4	G_5	G_6
G_1	1	0.737	0.717	0.714	0.709	0.729	1	0.72	0.718	0.716	0.71	0.709
G_2	0.737	1	0.703	0.702	0.7	0.718	0.72	1	0.71	0.708	0.702	0.703
G_3	0.717	0.703	1	0.703	0.7	0.717	0.718	0.71	1	0.705	0.701	0.703
G_4	0.714	0.702	0.703	1	0.702	0.718	0.716	0.708	0.705	1	0.7	0.702
G_5	0.709	0.7	0.7	0.702	1	0.725	0.71	0.702	0.701	0.7	1	0.703
G_6	0.729	0.718	0.717	0.718	0.725	1	0.709	0.703	0.703	0.702	0.703	1
	Manufacturing sector						Mining sector					
	G_1	G_2	G_3	G_4	G_5	G_6	G_1	G_2	G_3	G_4	G_5	G_6
G_1	1	0.714	0.713	0.723	0.708	0.705	1	0.742	0.712	0.716	0.714	0.703
G_2	0.714	1	0.709	0.718	0.704	0.702	0.742	1	0.7	0.704	0.702	0.7
G_3	0.713	0.709	1	0.718	0.705	0.704	0.712	0.7	1	0.706	0.703	0.7
G_4	0.723	0.718	0.718	1	0.7	0.7	0.716	0.704	0.706	1	0.705	0.7
G_5	0.708	0.704	0.705	0.7	1	0.7	0.714	0.702	0.703	0.705	1	0.7
G_6	0.705	0.702	0.704	0.7	0.7	1	0.703	0.7	0.7	0.7	0.7	1

Table 13. Degree of important of goals matrix using BON-IOWAIMAM

	Trading and consumer sector						Service sector					
	G ₁	G ₂	G ₃	G ₄	G ₅	G ₆	G ₁	G ₂	G ₃	G ₄	G ₅	G ₆
G ₁	1	0.852	0.862	0.86	0.855	0.867	1	0.858	0.854	0.859	0.854	0.858
G ₂	0.852	1	0.864	0.862	0.858	0.867	0.858	1	0.856	0.861	0.857	0.86
G ₃	0.862	0.864	1	0.862	0.859	0.867	0.854	0.856	1	0.864	0.857	0.857
G ₄	0.860	0.862	0.862	1	0.858	0.867	0.859	0.861	0.864	1	0.856	0.857
G ₅	0.855	0.858	0.859	0.858	1	0.874	0.854	0.857	0.857	0.856	1	0.86
G ₆	0.867	0.867	0.867	0.867	0.874	1	0.858	0.86	0.857	0.857	0.86	1
	Manufacturing sector						Mining sector					
	G ₁	G ₂	G ₃	G ₄	G ₅	G ₆	G ₁	G ₂	G ₃	G ₄	G ₅	G ₆
G ₁	1	0.86	0.856	0.857	0.853	0.859	1	0.842	0.853	0.849	0.854	0.841
G ₂	0.86	1	0.86	0.859	0.857	0.861	0.842	1	0.856	0.856	0.86	0.833
G ₃	0.856	0.86	1	0.861	0.857	0.861	0.853	0.856	1	0.854	0.86	0.835
G ₄	0.857	0.859	0.861	1	0.854	0.857	0.849	0.856	0.854	1	0.86	0.834
G ₅	0.853	0.857	0.857	0.854	1	0.86	0.854	0.86	0.86	0.86	1	0.834
G ₆	0.859	0.861	0.861	0.857	0.86	1	0.841	0.833	0.835	0.834	0.834	1

In service sector, it is noteworthy that goals with minimum difference are G₁ and G₆, where interrelationship existing between each goal is strong. It means that to ensure the achievement of goals guarantee continuity of operation and protect people, the other goals must be aligned. Goals with maximum difference are G₂, G₃ and G₄, where interrelationship between each goal is weak. It means that to ensure the achievement of these goals, it is not necessary to align another ones. In manufacturing sector, it is noteworthy that G₅ and G₆ are the goals with the least distance and the most distance between each goal is G₄, where interrelationship between goals is weak. It means that goals are align to get prevent economic losses and protect people and keep the good will is achieved by relationship between G₅ and G₆. Finally, in mining sector, it is noteworthy that there exists a greater interrelationship between each goal, which is reflected in G₃, G₄, G₅ and G₆ while a weaker interrelationship is reflected in G₁ and G₂. It means that in this sector great deals of goals are aligned to achieve, which denotes interdependency. Thus, using this method is observed that for all economic sectors, the goal with the greatest interrelationship for the other ones to be achieved is protect people.

For BON-IOWAIMAM results change considerably according to the maximum and minimum distance between each goal to achieve a specific goal. In this sense, it is observed that in commerce and consumer sector, goals with minimum distance are G₃, G₄ and G₆ while goal with maximum distance is G₅, which show a significant variation of the results. Likewise, in service sector, G₁, G₅ and G₆ have minimum distance and G₃ and G₄ have maximum distance. In manufacturing sector, G₂ and G₆ have minimum distance and G₁, G₃, G₄ and G₅ have maximum distance. Finally, in mining sector, G₆ have minimum distance and G₂, G₃, G₄ and G₅ have maximum distance. All results obtained using this method is opposite to

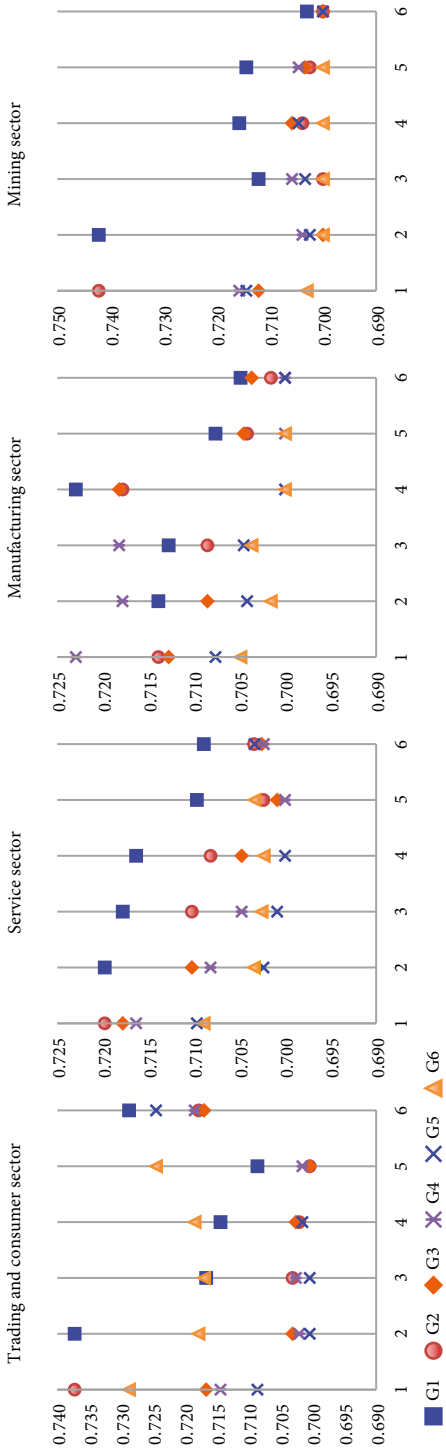


Figure 1. Degree of important of goals matrix using BON-IOWAAC

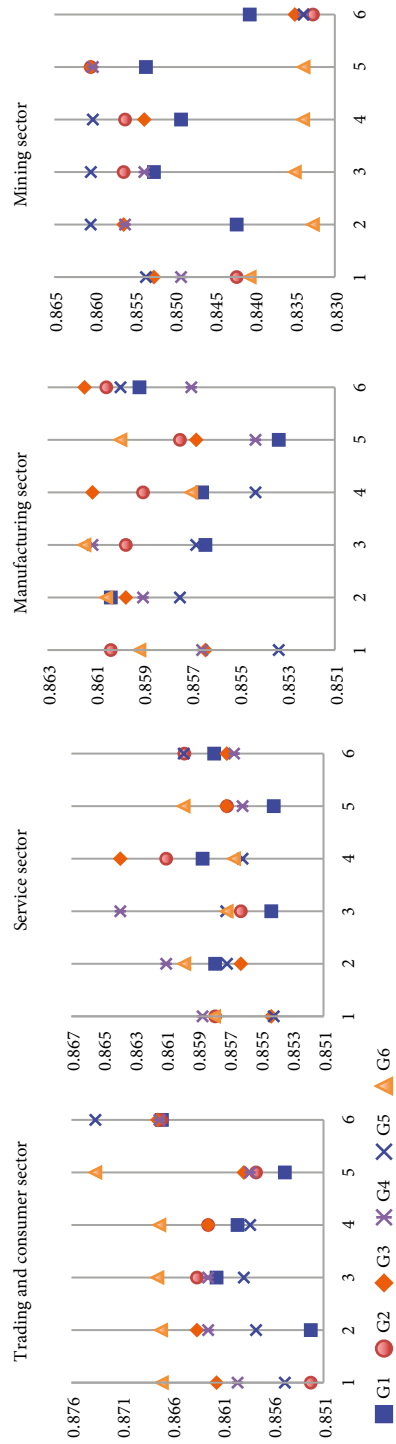


Figure 2. Degree of important of goals matrix using BON-IOWAIMAM

the previous method, since it is noted that there are more interrelationship and less aligned goals. For example, in commerce and consumer sector guarantee availability and quality of information, keep the good will and protect people are the goals with the greatest inter-relationship for the other ones to be achieved while prevent economic losses is less aligned.

Thus, it is noted that for each economic sector there is a variation of the goals with less alienation according to expectations of senior managers who lead ERM, i. e. results depends on the perception of the manager and not on the type of information. In this sense, it is important to highlight that these methods allowed the comparison and continuous relation of the specific variables (risk, benefits and objectives) for each sector. To conclude analysis result, we have presented a relationship between the most representative statistical and non-statistical outcomes (see Table 14).

On the one hand, the statistical analysis allowed doing a comprehensive interpretation of ERM in large companies in Colombia. This analysis shows in a descriptive way the degree of correlation of the study variables by the four determined categories (tools, commitment, minimization and areas). Likewise, from these variables, a multiple linear regression is derived that confirms the importance of the identification of the risks, from the monitoring, the control measures and the resources destined for an effective of the ERM. On the other hand, the non-parametric analyzes allowed to show the degree of importance of the goals and interrelationship according to the sorts of risk and the expected financial benefit for the sector. Thus, it can be concluded that the identification of risks is important for companies, but there are gradations according to the sector and the goals that each of them wants to achieve.

Table 14. Relationship between parametric and nonparametric results

Method	Analysis				
Correlation matrices results	According to the correlation matrices, it is observed that the variables with the highest correlation and level of significance for each category of the ERM are:				
	Categories	Tools	Commitment	Minimization	Areas
	Essential aspects	Use key indicators	Monitoring	Control measures	Necessary resources
Multiple linear regression results	The regression model is presented with the variables that had strong positive relationships for each of the correlation matrices of the ERM categories.				
	Identification of risks		the monitoring of risks for decision making (RMDM)	Control measures (MC)	Resources allocated (RA)
	Essential aspect		control measures (MC)		
Nonparametric results	With this analysis it was possible to see the distances and interrelation between the objectives to achieve a common goal and obtain an expected benefit, taking into account the perception of the managers of each sector:				
	Method		BON-IOWAAC		BON-IOWAIMAM
	Goal greatest interrelationship		Protect people		Protect people
	Thus, it is noted that for each economic sector there is a variation of the goals with less alienation according to expectations of senior managers who lead ERM, i.e. results depends on the perception of the manager and not on the type of information. In this sense, it is important to highlight that these methods allowed the comparison and continuous relation of the specific variables (risk, benefits and objectives) for each sector.				

Usually, in the analysis of the information, the statistical tools are the utilized and validated ones. However, these techniques are characterized by making a measurement of the information supplied within hard numerical parameters established without taking into account the meaning of that information (Blanco-Mesa, Merigó, & Gil-Lafuente, 2017). Within the literature it can find methodological approaches that allow data processing to search for patterns (Kochenderfer, 2015) and others that capture reasoning, semantics and attitude (Blanco-Mesa, Gil-Lafuente, & Merigo, 2018) for the treatment of risk and uncertainty. Thus, the development of hybrid methodologies (combining of soft and hard methods) provide solutions to information processing taking into account both its measurement and its meaning. In the case treated in this research, it is shown how both types of methods provide significant results in two different ways, the identification of the risks and the strategic thinking of the ERM. Thus, the essential aspect in the measurement of risks is determined while capturing the perception and opinion of the managers in the ERM.

It is noteworthy that this research propose nonparametric models, which allow the carrying out of subjective information analysis. They stand out for being able to aggregate information and highlight the importance of information. In addition, as a hybrid methodological proposal is consistent, which combines a statistical model (as is the Bonferroni means), a non-statistical model that represents the attitude (as IOWA operator) and mathematical measures that compare information (such as distance measurements AC and IMAM). Hence, the methodologies proposed and used in this research allow in a simple process to aggregate, compare, interrelate the information simultaneously. As a final result, it can be observed that the interrelation and difference between the objectives set through the different types of risk and the attitude of the manager (represented in the expected benefit) have been established. Finally, it is remarkable that this research has some limitations originated by instrument used, because it does not allow carrying out more robust and complex analyzes. Questions are structured in a mixed way where dichotomous and scale questions are found. These characteristics generate difficulties to create more robust constructs to carry out a structural equation model or regressions that have statistical consistency and can explain a model. Hence, the development of correlation matrices and a stratified multiple linear regression are proposed as a reliable statistical alternative for the processing and analysis of the obtained data.

Conclusions and discussion

In this paper, we have studied the main characteristics on ERM and identified the importance of ERM in large companies in Colombia using statistic and non-statistics analysis. Literature on risk management at the global and local levels is reviewed emphasizing on risk governance, practices and tools, communication and consultation and ERM development. Likewise, a literature review of decision-making and uncertainty is carried out to propose a new aggregation method. We have used a dataset that enclose ERM information in Colombia to determine if ERM is important for large companies in relation to the identification, evaluation, monitoring and control of risks. Furthermore, the degree of importance of the goals and their interrelationship with expected financial benefits and the sorts of risks in ERM is determined as well.

For parametric analysis, the identification of risks within organizations, the establishment and use of key indicators are fundamental for their monitoring. In addition, there is also evidence of a strong relationship between risk indicators as an effective measure of application for the treatment of these. Also, risk event register is one of the most relevant tools to take control measures in the management of risks. Finally, culture has a significant relationship with the necessary resources that can be assigned to risk management. However, this culture is not influenced by the existence of a specific area that coordinates risk management within organizations. Furthermore, the identification of risks at the corporate level is explained by the relationship with the monitoring of risks for decision making (RMDM), control measures (CM) and resources allocated (RA), where CM is highlighted by the businessmen use different tools use different tools to measure and control risks.

For nonparametric analysis, we have proposed a new aggregation operators combining BON-IOWA operator with some distance measures, which are called the Bonferroni Induced Ordered Weighted Average Adequacy Coefficient (BON-IHOWAAC) operator and Bonferroni Induced Ordered Weighted Average the Maximum and Minimum level (BON-IHOW-AIMAM). The main advantage of these new operators is that can calculate the differences between two elements or two sets and reorder the arguments using order-inducing variables in order to solve more complex problems. Based on these methods, the risk management information and perception of the managers are used to compare goals in ERM through the different sorts of risks as main characteristics and expected financial benefit as weighted vectors for four main economic sectors. Of the results obtained is highlighted that for all economic sectors, the goal with the greatest interrelationship for the other ones to be achieved is protect people. Furthermore, it is noted that for each economic sector there is a variation of the goals with less alienation according to senior manager expectations who lead ERM, i.e. results depends on manager perceptions and not the sort of information. Hence, with these both analyses allow observing the importance that give managers at ERM in a holistic manner. Managers find in ERM a fundamental strategy in the design and structuring of organizational planning, because it enable to identify, evaluate and control the most relevant risks from a holistic viewpoint. In addition, these analyzes allow observing the perceptions that managers have in the management of risks, around an increasingly uncertain environment, with respect to the objectives that are proposed and the benefits they want to achieve by managing them.

It is important to point out that these results are given by the knowledge, attitudes and expectations in the decision making of the managers of the large companies in Colombia of the different sectors. It is noteworthy that decision makers do not always have a rational behavior and sometimes they and act compulsively by emotions and their own beliefs according to certain information received and the pressures of the environment. Thus, in this mathematical application, the distance reflects the differences between the information of each risk according to each goal. Therefore, it is possible to observe that there are goals that are prioritized to achieve the expected benefit.

With these results, we have showed that statistical methods could be efficient in the analysis and prediction of events. However, in many cases, it is necessary to resort to other techniques that can also help managers to make decisions based on their preferences in an

orderly and logical manner. Also, it is noteworthy that the results allow reviewing objective and subjective information from different sources obtained representative values, which provide a global view of the environment in these sectors. Additionally, it allows dealing with uncertainty beyond the importance of the characteristics of the information. Based on this study, companies can improve their market position, help them to keep up over time, improve their decision-making, which will help the general economy of Colombia by having more and better companies that can administer their risk in a more appropriate way, focusing their efforts on the right actions and that are not rambling. Finally, this study gives way to new research aimed at analyzing various methods, which provide a new holistic view of how enterprise risks are managed, through the perceptions and opinions of managers in different economic sectors at the national and international levels generating new scientific knowledge.

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All the authors included in this article declare that they do not have any competing financial, professional, or personal interests from other parties.

References

- Ai, J., Brockett, P. L., & Wang, T. (2017). Optimal enterprise risk management and decision making with shared and dependent risks. *Journal of Risk and Insurance*, 84(4), 1127-1169. <https://doi.org/10.1111/jori.12140>
- Arnold, V., Benford, T., Canada, J., & Sutton, S. G. (2015). Leveraging integrated information systems to enhance strategic flexibility and performance: The enabling role of enterprise risk management. *International Journal of Accounting Information Systems*, 19, 1-16. <https://doi.org/10.1016/j.accinf.2015.10.001>
- AS/NZS-4360. (2004). *Australian/New Zealand standard risk management* (3rd ed.). Sydney, Australia: Standards Australia and Standards New Zealand.
- Beasley, M. S., Clune, R., & Hermanson, D. R. (2005). Enterprise risk management: An empirical analysis of factors associated with the extent of implementation. *Journal of Accounting and Public Policy*, 24(6), 521-531. <https://doi.org/10.1016/j.jaccpubpol.2005.10.001>
- Beliakov, G., Bustince Sola, H., & Calvo Sánchez, T. (2016). *A practical guide to averaging functions*. Switzerland: Springer International Publishing. <https://doi.org/10.1007/978-3-319-24753-3>
- Beliakov, G., James, S., Mordelová, J., Rückschlossová, T., & Yager, R. R. (2010). Generalized Bonferroni mean operators in multi-criteria aggregation. *Fuzzy Sets and Systems*, 161(17), 2227-2242. <https://doi.org/10.1016/j.fss.2010.04.004>
- Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management Science*, 17(4), B-141-B-164. <https://doi.org/10.1287/mnsc.17.4.B141>

- Bill, A. (2006). Entendiendo la administración de riesgo empresarial. Un modelo emergente para generar valor al accionista. In *Swiss Re new markets* (pp. 1-27). Retrieved from <https://www.riesgooperacional.com/docs/19%20ERM%20Paper.pdf>
- Blanco-Mesa, F. (2015). *Técnicas para la toma de decisiones en contextos inciertos: Identificación de oportunidades socio-económicas en el ámbito deportivo*. Universitat de Barcelona. Retrieved from <http://www.tesisenred.net/handle/10803/322786>
- Blanco-Mesa, F., León-Castro, E., & Merigó, J. M. (2018a). Bonferroni induced heavy operators in ERM decision-making: A case on large companies in Colombia. *Applied Soft Computing*, 72, 371-391. <https://doi.org/10.1016/j.asoc.2018.08.001>
- Blanco-Mesa, F., León-Castro, E., Merigó, J. M., & Xu, Z. (2018b). Bonferroni means with induced ordered weighted average operators. *International Journal of Intelligent Systems*, 34(1), 3-23. <https://doi.org/10.1002/int.22033>
- Blanco-Mesa, F., & Merigo-Lindahl, J. M. (2016). Bonferroni distances with OWA operators. In *2016 Annual Conference of the North American Fuzzy Information Processing Society (NAFIPS)* (pp. 1-5). IEEE. <https://doi.org/10.1109/NAFIPS.2016.7851586>
- Blanco-Mesa, F., & Merigó, J. M. (2016). Bonferroni means with the adequacy coefficient and the index of maximum and minimum level. *Lecture Notes in Business Information Processing*, 254, 155-166. https://doi.org/10.1007/978-3-319-40506-3_16
- Blanco-Mesa, F., & Merigó, J. M. (2017). Bonferroni distances with hybrid weighted distance and immediate weighted distance. *Fuzzy Economic Review*, 22(2), 29-43. <https://doi.org/10.25102/fer.2017.02.02>
- Blanco-Mesa, F., Merigó, J. M., & Gil-Lafuente, A. M. (2017). Fuzzy decision making: A bibliometric-based review. *Journal of Intelligent & Fuzzy Systems*, 32(3), 2033-2050. <https://doi.org/10.3233/JIFS-161640>
- Blanco-Mesa, F., Merigó, J. M., & Kacprzyk, J. (2016). Bonferroni means with distance measures and the adequacy coefficient in entrepreneurial group theory. *Knowledge-Based Systems*, 111(1), 217-227. <https://doi.org/10.1016/j.knosys.2016.08.016>
- Blanco-Mesa, F., Gil-Lafuente, A. M., & Merigo, J. M. (2018). Dynamics of stakeholder relations with multi-person aggregation. *Kybernetes*, 47(9), 1801-1820. <https://doi.org/10.1108/K-11-2017-0454>
- Bonferroni, C. (1950). Sulle medie multiple di potenze. *Bollettino Dell'Unione Matematica Italiana*, 5(3-4), 267-270.
- Bromiley, P., McShane, M., Nair, A., & Rustambekov, E. (2015). Enterprise risk management: Review, critique, and research directions. *Long Range Planning*, 48(4), 265-276. <https://doi.org/10.1016/j.lrp.2014.07.005>
- Casanovas, M., Merigó, J. M., & Torres-Martínez, A. (2014). *Inteligencia computacional en la gestión del riesgo asegurador: Operadores de agregación OWA en procesos de tarificación*. Madrid: Fundación MAPFRE.
- Casualty Actuarial Society. (2003). *Overview of enterprise risk management*. Retrieved from <https://www.casact.org/area/erm/overview.pdf>
- Cifuentes, J., Blanco-Mesa, F., Gutiérrez, L. G., Oviedo, E., Godoy, M. R., & Mora, J. M. (2017). Diagnóstico del desarrollo en seguridad y salud laboral en grandes empresas privadas de Bogotá, Colombia. In R. C. Mejía, M. A. Núñez, & I. Martínez (Eds.), *Administración de riesgos empresariales* (Primera ed., p. 24). Bogotá, Colombia: Editorial EAFIT, Uduál, Alafec.
- Correa, G., Rios, E., & Acevedo, J. (2017). Evolución de la cultura de la gestión de riesgos en el entorno empresarial Colombiano. *Journal of Engineering and Technology*, 6(1), 22-45. <https://doi.org/10.22507/jet.v6n1a2>
- COSO. (2004). *Enterprise risk management: Integrated framework*. Committee of Sponsoring Organizations of the Treadway Commission. Retrieved from http://www.coso.org/documents/coso_erm_executivesummary.pdf

- COSO. (2017). *Enterprise risk management: Integrating with strategy and performance*. Committee of Sponsoring Organizations of the Treadway Commission.
- Davidson, O., Mackenzie, P., Wilkinson, M., & Burke, R. (2015). *Risk culture*. In P. Green (Ed.), *Enterprise risk management: A common framework for the entire organization* (1th ed, pp. 177-193). Elsevier Inc.
- Deloitte, Touche, & IMEF. (2003). *Administración integral de riesgos de negocio*. México: Instituto Mexicano de Ejecutivos de Finanzas, A.C.
- Demidenko, E., & McNutt, P. (2010). The ethics of enterprise risk management as a key component of corporate governance. *International Journal of Social Economics*, 37(10), 802-815. <https://doi.org/10.1108/03068291011070462>
- Dutta, B., & Guha, D. (2013). Trapezoidal intuitionistic fuzzy Bonferroni means and its application in multi-attribute decision making. In *2013 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)* (pp. 1-8). IEEE. <https://doi.org/10.1109/FUZZ-IEEE.2013.6622367>
- Dutta, B., & Guha, D. (2015). Partitioned Bonferroni mean based on linguistic 2-tuple for dealing with multi-attribute group decision making. *Applied Soft Computing*, 37, 166-179. <https://doi.org/10.1016/j.asoc.2015.08.017>
- Gantz, S. D., & Philpott, D. R. (2013). Success factors. In *FISMA and the risk management framework* (pp. 105-129) Elsevier. <https://doi.org/10.1016/B978-1-59-749641-4.00005-9>
- Gil-Lafuente, J. (2002). *Keys for success in sport management*. Vigo: Ed. Malladoiro.
- Gong, Y., Hu, N., Zhang, J., Liu, G., & Deng, J. (2015). Multi-attribute group decision making method based on geometric Bonferroni mean operator of trapezoidal interval type-2 fuzzy numbers. *Computers & Industrial Engineering*, 81, 167-176. <https://doi.org/10.1016/j.cie.2014.12.030>
- Hamming, R. W. (1950). Error detecting and error correcting codes. *Bell System Technical Journal*, 29(2), 147-160. <https://doi.org/10.1002/j.1538-7305.1950.tb00463.x>
- HB 327. (2010). *Communicating and consulting about risk* (4th ed.). Sydney, Australia: Standards Australia Limited.
- He, Y., & He, Z. (2015). Extensions of Atanassov's intuitionistic fuzzy interaction Bonferroni means and their application to multiple attribute decision making. *IEEE Transactions on Fuzzy Systems*, PP(99), 1. <https://doi.org/10.1109/TFUZZ.2014.2372074>
- Herrera, F., & Herrera-Viedma, E. (2000). Linguistic decision analysis: Steps for solving decision problems under linguistic information. *Fuzzy Sets and Systems*, 115(1), 67-82. [https://doi.org/10.1016/S0165-0114\(99\)00024-X](https://doi.org/10.1016/S0165-0114(99)00024-X)
- Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1996). Direct approach processes in group decision making using linguistic OWA operators. *Fuzzy Sets and Systems*, 79(2), 175-190. [https://doi.org/10.1016/0165-0114\(95\)00162-X](https://doi.org/10.1016/0165-0114(95)00162-X)
- Herrera, F., & Martínez, L. (2000). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, 8(6), 746-752. <https://doi.org/10.1109/91.890332>
- Hillson, D. A. (1997). Towards a risk maturity model. *The International Journal of Project and Business Risk Management*, 1, 35-45.
- ICONTEC. (2009). *Gestión del riesgo. Principios y directrices*. Colombia.
- IRM, & Protiviti. (2012). *Risk culture under the microscope guidance for boards*. Londres, Inglaterra.
- ISO. (2009a). *Risk management: Principles and guidelines*. Suiza: International Organization for Standardization.
- ISO. (2009b). *Risk management: Vocabulary* (Pub. L. No. 73, 15). Suiza: International Organization for Standardization.
- Jiang, X. P., & Wei, G. W. (2014). Some Bonferroni mean operators with 2-tuple linguistic information and their application to multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology*, 27(5), 2153-2162.

- Kaufmann, A., & Gil Aluja, J. (1987). *Técnicas operativas de gestión para el tratamiento de la incertidumbre*. Barcelona: Hispano Europea.
- Kaufmann, A., & Gil Aluja, J. (1986). *Introducción de la teoría de los suconjuntos borrosos a la gestión de las empresas* (2nd ed.). Santiago de Compostela: Milladoiro.
- Kochenderfer, M. J. (2015). *Decision making under uncertainty: Theory and application*. Cambridge, MA.: MIT Lincoln Laboratory Series.
- Kopia, J., Just, V., Geldmacher, W., & Bubian, A. (2017). Organization performance and enterprise risk management. *Ecoforum*, 6(1(10)).
- Li, D., Zeng, W., & Li, J. (2016). Geometric Bonferroni mean operators. *International Journal of Intelligent Systems*, 31(12), 1181–1197. <https://doi.org/10.1002/int.21822>
- Liu, H. C., Mao, L.-X., Zhang, Z.-Y., & Li, P. (2013). Induced aggregation operators in the VIKOR method and its application in material selection. *Applied Mathematical Modelling*, 37(9), 6325–6338. <https://doi.org/10.1016/j.apm.2013.01.026>
- Liu, P., Rong, L., Chu, Y., & Li, Y. (2014). Intuitionistic linguistic weighted Bonferroni mean operator and its application to multiple attribute decision making. *The Scientific World Journal*, 1-13. <https://doi.org/10.1155/2014/545049>
- Marc, M., Sprcic, D. M., D., & Žagar, M. M. (2018). Is enterprise risk management a value added activity? *E & M Ekonomija a Management*, 21(1), 68-84. <https://doi.org/10.15240/tul/001/2018-1-005>
- Mejía, R. C. (2013). *Identificación de riesgos empresariales*. Medellín, Colombia: Editorial EAFIT, Uduál, Alafec.
- Mejía, R. C. (2017). Introducción. In R. C. Mejía, M. A. Núñez, & I. Martínez (Eds.), *Administración de riesgos empresariales* (primera, pp. 15-28). Medellín, Colombia: Editorial EAFIT, Uduál, Alafec.
- Mejía, R. C., Núñez, M. A., & Martínez, I. (Eds.). (2017). *Administración de riesgos empresariales*. Medellín, Colombia: Editorial EAFIT, Uduál, Alafec.
- Merigó, J. M. (2010). Fuzzy decision making with immediate probabilities. *Computers and Industrial Engineering*, 58(4), 651-657. <https://doi.org/10.1016/j.cie.2010.01.007>
- Merigó, J. M. (2014). *La media ponderada ordenada probabilística: Teoría y aplicaciones* (Collecció Reial acadèmia de doctores). Barcelona España: Fundació Universitaria ESERP.
- Merigó, J. M., & Casanovas, M. (2011). Decision-making with distance measures and induced aggregation operators. *Computers & Industrial Engineering*, 60(1), 66-76. <https://doi.org/10.1016/j.cie.2010.09.017>
- Merigó, J. M., & Gil-Lafuente, A. M. (2008). On the use of the OWA operator in the Euclidean distance. *International Journal of Computer Science and Engineering*, 2(4), 170-176.
- Merigó, J. M., & Gil-Lafuente, A. M. (2009). The induced generalized OWA operator. *Information Sciences*, 179(6), 729-741. <https://doi.org/10.1016/j.ins.2008.11.013>
- Merigó, J. M., & Gil-Lafuente, A. M. (2012). Decision making techniques with similarity measures and OWA operators. *SORT – Statistics and Operations Research Transactions*, 36(1), 81-102.
- Merigó, J. M., Palacios-Marqués, D., & Soto-Acosta, P. (2017). Distance measures, weighted averages, OWA operators and Bonferroni means. *Applied Soft Computing*, 50, 356-366. <https://doi.org/10.1016/j.asoc.2016.11.024>
- Merigó, J. M., & Wei, G. (2011). Probabilistic aggregation operators and their application in uncertain multi-person decision-making. *Technological and Economic Development of Economy*, 17(2), 335-351. <https://doi.org/10.3846/20294913.2011.584961>
- Merigó, J. M., & Yager, R. R. (2013). Generalized moving average, distance measures and OWA operators. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 21(04), 533-559. <https://doi.org/10.1142/S0218488513500268>

- Modave, F., Ceberio, M., & Kreinovich, V. (2008). *Choquet integrals and OWA criteria as a natural (and optimal) next step after linear aggregation: A new general justification* (pp. 741-753). Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-540-88636-5_70
- OECD. (2014). *Risk management and corporate governance*. OECD Publishing.
- Quon, T. K., Zeghal, D., & Maingot, M. (2012). Enterprise risk management and firm performance. *Procedia - Social and Behavioral Sciences*, 62, 263-267. <https://doi.org/10.1016/j.sbspro.2012.09.042>
- RIMS. (2006). *RIMS Risk maturity model (RMM) for enterprise risk management preface and history*. Risk and Insurance Management Society, Inc.
- Scarlat, E., Chirita, N., & Bradea, I.A. (2012). Indicators and metrics used in the enterprise risk management (ERM). *Economic Computation and Economic Cybernetics Studies and Research*, 46, 5-18.
- Thomas, R., Berry, S., & Jianren, X. (2018). Enterprise risk management and the cost of capital. *The Journal of Risk and Insurance*, 85(1), 159-201. <https://doi.org/10.1111/jori.12152>
- Wang, C., Li, Q., Zhou, X., & Yang, T. (2014). Hesitant triangular fuzzy information aggregation operators based on Bonferroni means and their application to multiple attribute decision making. *The Scientific World Journal*, 1-15. <https://doi.org/10.1155/2014/648516>
- Wei, G., Zhao, X., Lin, R., & Wang, H. (2013). Uncertain linguistic Bonferroni mean operators and their application to multiple attribute decision making. *Applied Mathematical Modelling*, 37(7), 5277-5285. <https://doi.org/10.1016/j.apm.2012.10.048>
- Wu, D., Olson, D. L., & Dolgui, A. (2015). Decision making in enterprise risk management: A review and introduction to special issue. *Omega*, 57, 1-4. <https://doi.org/10.1016/j.omega.2015.04.011>
- Xia, M., Xu, Z., & Zhu, B. (2013). Geometric Bonferroni means with their application in multi-criteria decision making. *Knowledge-Based Systems*, 40, 88-100. <https://doi.org/10.1016/j.knosys.2012.11.013>
- Xu, Z. (2012). Uncertain Bonferroni mean operators. *International Journal of Computational Intelligence Systems*, 3(6), 761-769. <https://doi.org/10.1080/18756891.2010.9727739>
- Xu, Z. S., & Da, Q. L. (2002). The uncertain OWA operator. *International Journal of Intelligent Systems*, 17(6), 569-575. <https://doi.org/10.1002/int.10038>
- Xu, Z., & Yager, R. R. (2011). Intuitionistic fuzzy Bonferroni means. *IEEE Transactions on Systems, Man, and Cybernetics. Part B, Cybernetics: A Publication of the IEEE Systems, Man, and Cybernetics Society*, 41(2), 568-578. <https://doi.org/10.1109/TSMCB.2010.2072918>
- Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multicriteria decision-making. *IEEE Transactions on Systems, Man, and Cybernetics*, 18(1), 183-190. <https://doi.org/10.1109/21.87068>
- Yager, R. R. (1993). Families of OWA operators. *Fuzzy Sets and Systems*, 59(2), 125-148. [https://doi.org/10.1016/0165-0114\(93\)90194-M](https://doi.org/10.1016/0165-0114(93)90194-M)
- Yager, R. R. (2002). Heavy OWA operators. *Fuzzy Optimization and Decision Making*, 1(4), 379-397. <https://doi.org/10.1023/A:1020959313432>
- Yager, R. R. (2003a). E-Z OWA weights. In *10th IFSA World Congress* (pp. 39-42). Istanbul.
- Yager, R. R. (2003b). Induced aggregation operators. *Fuzzy Sets and Systems*, 137(1), 59-69. [https://doi.org/10.1016/S0165-0114\(02\)00432-3](https://doi.org/10.1016/S0165-0114(02)00432-3)
- Yager, R. R. (2008). Time series smoothing and OWA aggregation. *IEEE Transactions on Fuzzy Systems*, 16(4), 994-1007. <https://doi.org/10.1109/TFUZZ.2008.917299>
- Yager, R. R. (2009). On generalized Bonferroni mean operators for multi-criteria aggregation. *International Journal of Approximate Reasoning*, 50(8), 1279-1286. <https://doi.org/10.1016/j.ijar.2009.06.004>
- Yager, R. R. (2014). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, 22(4), 958-965. <https://doi.org/10.1109/TFUZZ.2013.2278989>

- Yager, R. R., & Filev, D. P. (1999). Induced ordered weighted averaging operators. *IEEE Transactions on Systems, Man, and Cybernetics. Part B, Cybernetics: A Publication of the IEEE Systems, Man, and Cybernetics Society*, 29(2), 141-150. <https://doi.org/10.1109/3477.752789>
- Yager, R. R., Kacprzyk, J., & Beliakov, G. (2011). *Recent developments in the ordered weighted averaging operators: Theory and practice. Studies in Fuzziness and Soft Computing* (Vol. 265). Berlin, Heidelberg: Springer Science & Business Media. <https://doi.org/10.1007/978-3-642-17910-5>
- Yu, D., Wu, Y., & Zhou, W. (2012). Generalized hesitant fuzzy Bonferroni mean and its application in multi-criteria group decision making. *Journal of Information & Computational Science*, 9(2), 267-274.
- Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338-353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- Zadeh, L. A. (1975a). The concept of a linguistic variable and its application to approximate reasoning I. *Information Sciences*, 8, 199-249. [https://doi.org/10.1016/0020-0255\(75\)90036-5](https://doi.org/10.1016/0020-0255(75)90036-5)
- Zadeh, L. A. (1975b). The concept of a linguistic variable and its application to approximate reasoning II. *Information Sciences*, 8(4), 301-357. [https://doi.org/10.1016/0020-0255\(75\)90046-8](https://doi.org/10.1016/0020-0255(75)90046-8)
- Zadeh, L. A. (1975c). The concept of a linguistic variable and its application to approximate reasoning III. *Information Sciences*, 9(1), 43-80. [https://doi.org/10.1016/0020-0255\(75\)90017-1](https://doi.org/10.1016/0020-0255(75)90017-1)
- Zeng, S., Su, W., & Zhang, C. (2016). Intuitionistic fuzzy generalized probabilistic ordered weighted averaging operator and its application to group decision making. *Technological and Economic Development of Economy*, 22(2), 177-193. <https://doi.org/10.3846/20294913.2014.984253>
- Zhu, B., & Xu, Z. S. (2013). Hesitant fuzzy Bonferroni means for multi-criteria decision making. *Journal of the Operational Research Society*, 64(12), 1831-1840. <https://doi.org/10.1057/jors.2013.7>
- Zhu, B., Xu, Z., & Xia, M. (2012). Hesitant fuzzy geometric Bonferroni means. *Information Sciences*, 205, 72-85. <https://doi.org/10.1016/j.ins.2012.01.048>